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MATHEMATICAL TABLES

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FIRST SERIES

BY

JAMES MILLS PEIRCE

UNIVERSITY PROFESSOR OF MATHEMATICS IN HARVARD UNIVERSITY

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185	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
186	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
187	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
188	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
189	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
190	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
191	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
192	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
193	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
194	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
195	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
196	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
197	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
198	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
199	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

A	6.	7.	8.	9.	0.	1.	2.	3.
00	0.0000	0.0004	0.0043	0.0414 9	0.3010 50	1.0414 91	2.0043	3.0004
01	0.0000	0.0004	0.0044	0.0423 9	0.3061 51	1.0505 91	2.0142	3.0104
02	0.0000	0.0005	0.0045	0.0432 9	0.3111 51	1.0596 91	2.0241	3.0204
03	0.0000	0.0005	0.0046	0.0442 10	0.3163 52	1.0687 91	2.0340	3.0304
04	0.0000	0.0005	0.0047	0.0452 10	0.3215 52	1.0779 92	2.0439	3.0404
05	0.0000	0.0005	0.0048	0.0462 10	0.3267 53	1.0871 92	2.0539	3.0504
06	0.0000	0.0005	0.0050	0.0472 10	0.3321 53	1.0963 92	2.0638	3.0604
07	0.0001	0.0005	0.0051	0.0482 11	0.3374 54	1.1055 92	2.0737	3.0704
08	0.0001	0.0005	0.0052	0.0493 11	0.3429 55	1.1147 92	2.0836	3.0804
09	0.0001	0.0005	0.0053	0.0504 11	0.3484 55	1.1239 92	2.0935	3.0904
10	0.0001	0.0005	0.0054	0.0515 11	0.3539 56	1.1332 93	2.1034	3.1003
11	0.0001	0.0006	0.0056	0.0526 11	0.3595 56	1.1425 93	2.1134	3.1103
12	0.0001	0.0006	0.0057	0.0538 12	0.3652 57	1.1518 93	2.1233	3.1203
13	0.0001	0.0006	0.0058	0.0550 12	0.3709 57	1.1611 93	2.1332	3.1303
14	0.0001	0.0006	0.0060	0.0562 12	0.3766 58	1.1704 93	2.1431	3.1403
15	0.0001	0.0006	0.0061	0.0574 12	0.3825 59	1.1797 93	2.1531	3.1503
16	0.0001	0.0006	0.0062	0.0586 13	0.3884 59	1.1891 94	2.1630	3.1603
17	0.0001	0.0006	0.0064	0.0599 13	0.3943 60	1.1984 94	2.1729	3.1703
18	0.0001	0.0007	0.0065	0.0612 13	0.4003 60	1.2078 94	2.1829	3.1803
19	0.0001	0.0007	0.0067	0.0625 13	0.4063 61	1.2172 94	2.1928	3.1903
20	0.0001	0.0007	0.0068	0.0639 14	0.4124 61	1.2266 94	2.2027	3.2003
21	0.0001	0.0007	0.0070	0.0653 14	0.4186 62	1.2360 94	2.2127	3.2103
22	0.0001	0.0007	0.0071	0.0667 14	0.4248 62	1.2454 94	2.2226	3.2203
23	0.0001	0.0007	0.0073	0.0681 15	0.4311 63	1.2548 94	2.2325	3.2303
24	0.0001	0.0008	0.0075	0.0696 15	0.4374 63	1.2643 95	2.2425	3.2402
25	0.0001	0.0008	0.0077	0.0711 15	0.4438 64	1.2738 95	2.2524	3.2502
26	0.0001	0.0008	0.0078	0.0726 15	0.4502 65	1.2832 95	2.2624	3.2602
27	0.0001	0.0008	0.0080	0.0742 16	0.4567 65	1.2927 95	2.2723	3.2702
28	0.0001	0.0008	0.0082	0.0757 16	0.4632 66	1.3022 95	2.2823	3.2802
29	0.0001	0.0008	0.0084	0.0774 16	0.4698 66	1.3117 95	2.2922	3.2902
30	0.0001	0.0009	0.0086	0.0790 17	0.4764 67	1.3212 95	2.3022	3.3002
31	0.0001	0.0009	0.0088	0.0807 17	0.4831 67	1.3308 95	2.3121	3.3102
32	0.0001	0.0009	0.0090	0.0824 17	0.4899 68	1.3403 95	2.3221	3.3202
33	0.0001	0.0009	0.0092	0.0841 18	0.4966 68	1.3499 96	2.3320	3.3302
34	0.0001	0.0009	0.0094	0.0859 18	0.5035 69	1.3594 96	2.3420	3.3402
35	0.0001	0.0010	0.0096	0.0877 18	0.5104 69	1.3690 96	2.3519	3.3502
36	0.0001	0.0010	0.0098	0.0896 19	0.5173 70	1.3786 96	2.3619	3.3602
37	0.0001	0.0010	0.0101	0.0915 19	0.5243 70	1.3881 96	2.3718	3.3702
38	0.0001	0.0010	0.0103	0.0934 19	0.5313 71	1.3977 96	2.3818	3.3802
39	0.0001	0.0011	0.0105	0.0953 20	0.5384 71	1.4073 96	2.3918	3.3902
40	0.0001	0.0011	0.0108	0.0973 20	0.5455 72	1.4170 96	2.4017	3.4002
41	0.0001	0.0011	0.0110	0.0993 20	0.5527 72	1.4266 96	2.4117	3.4102
42	0.0001	0.0011	0.0113	0.1014 21	0.5599 72	1.4362 96	2.4216	3.4202
43	0.0001	0.0012	0.0115	0.1035 21	0.5672 73	1.4458 96	2.4316	3.4302
44	0.0001	0.0012	0.0118	0.1057 22	0.5745 73	1.4555 96	2.4416	3.4402
45	0.0001	0.0012	0.0121	0.1078 22	0.5819 74	1.4651 97	2.4515	3.4502
46	0.0001	0.0013	0.0123	0.1101 22	0.5893 74	1.4748 97	2.4615	3.4602
47	0.0001	0.0013	0.0126	0.1123 23	0.5967 75	1.4845 97	2.4715	3.4701
48	0.0001	0.0013	0.0129	0.1146 23	0.6042 75	1.4941 97	2.4814	3.4801
49	0.0001	0.0013	0.0132	0.1169 24	0.6118 76	1.5038 97	2.4914	3.4901
50	0.0001	0.0014	0.0135	0.1193 24	0.6193 76	1.5135 97	2.5014	3.5001

A	6.	7.	8.	9.	0.	1.	2.	3.
50	0.0001	0.0014	0.0135	0.1193 24	0.6193 76	1.5135 97	2.5014	3.5001
51	0.0001	0.0014	0.0138	0.1218 24	0.6269 76	1.5232 97	2.5113	3.5101
52	0.0001	0.0014	0.0141	0.1242 25	0.6346 77	1.5329 97	2.5213	3.5201
53	0.0001	0.0015	0.0145	0.1267 25	0.6423 77	1.5426 97	2.5313	3.5301
54	0.0002	0.0015	0.0148	0.1293 26	0.6501 78	1.5523 97	2.5413	3.5401
55	0.0002	0.0015	0.0151	0.1319 26	0.6578 78	1.5621 97	2.5512	3.5501
56	0.0002	0.0016	0.0155	0.1345 27	0.6657 78	1.5718 97	2.5612	3.5601
57	0.0002	0.0016	0.0158	0.1372 27	0.6735 79	1.5815 97	2.5712	3.5701
58	0.0002	0.0016	0.0162	0.1399 28	0.6814 79	1.5913 97	2.5811	3.5801
59	0.0002	0.0017	0.0166	0.1427 28	0.6893 80	1.6010 97	2.5911	3.5901
60	0.0002	0.0017	0.0170	0.1455 28	0.6973 80	1.6108 98	2.6011	3.6001
61	0.0002	0.0018	0.0173	0.1484 29	0.7053 80	1.6205 98	2.6111	3.6101
62	0.0002	0.0018	0.0177	0.1513 29	0.7134 81	1.6303 98	2.6210	3.6201
63	0.0002	0.0018	0.0181	0.1543 30	0.7215 81	1.6401 98	2.6310	3.6301
64	0.0002	0.0019	0.0186	0.1573 30	0.7296 81	1.6498 98	2.6410	3.6401
65	0.0002	0.0019	0.0190	0.1604 31	0.7377 82	1.6596 98	2.6510	3.6501
66	0.0002	0.0020	0.0194	0.1635 31	0.7459 82	1.6694 98	2.6609	3.6601
67	0.0002	0.0020	0.0199	0.1666 32	0.7541 82	1.6792 98	2.6709	3.6701
68	0.0002	0.0021	0.0203	0.1699 32	0.7624 83	1.6890 98	2.6809	3.6801
69	0.0002	0.0021	0.0208	0.1731 33	0.7707 83	1.6988 98	2.6909	3.6901
70	0.0002	0.0022	0.0212	0.1764 33	0.7790 83	1.7086 98	2.7009	3.7001
71	0.0002	0.0022	0.0217	0.1798 34	0.7874 84	1.7184 98	2.7108	3.7101
72	0.0002	0.0023	0.0222	0.1832 34	0.7957 84	1.7282 98	2.7208	3.7201
73	0.0002	0.0023	0.0227	0.1867 35	0.8042 84	1.7380 98	2.7308	3.7301
74	0.0002	0.0024	0.0232	0.1902 35	0.8126 85	1.7478 98	2.7408	3.7401
75	0.0002	0.0024	0.0238	0.1938 36	0.8211 85	1.7577 98	2.7508	3.7501
76	0.0002	0.0025	0.0243	0.1974 37	0.8296 85	1.7675 98	2.7608	3.7601
77	0.0003	0.0025	0.0248	0.2011 37	0.8381 85	1.7773 98	2.7707	3.7701
78	0.0003	0.0026	0.0254	0.2048 38	0.8467 86	1.7871 98	2.7807	3.7801
79	0.0003	0.0027	0.0260	0.2086 38	0.8553 86	1.7970 98	2.7907	3.7901
80	0.0003	0.0027	0.0266	0.2124 39	0.8639 86	1.8068 98	2.8007	3.8001
81	0.0003	0.0028	0.0272	0.2163 39	0.8725 87	1.8167 98	2.8107	3.8101
82	0.0003	0.0029	0.0278	0.2203 40	0.8812 87	1.8265 99	2.8207	3.8201
83	0.0003	0.0029	0.0284	0.2243 40	0.8899 87	1.8364 99	2.8306	3.8301
84	0.0003	0.0030	0.0291	0.2284 41	0.8986 87	1.8462 99	2.8406	3.8401
85	0.0003	0.0031	0.0297	0.2325 41	0.9074 88	1.8561 99	2.8506	3.8501
86	0.0003	0.0031	0.0304	0.2366 42	0.9162 88	1.8660 99	2.8606	3.8601
87	0.0003	0.0032	0.0311	0.2409 43	0.9250 88	1.8758 99	2.8706	3.8701
88	0.0003	0.0033	0.0318	0.2452 43	0.9338 88	1.8857 99	2.8806	3.8801
89	0.0003	0.0034	0.0325	0.2495 44	0.9426 89	1.8956 99	2.8906	3.8901
90	0.0003	0.0034	0.0332	0.2539 44	0.9515 89	1.9054 99	2.9005	3.9001
91	0.0004	0.0035	0.0339	0.2584 45	0.9604 89	1.9153 99	2.9105	3.9101
92	0.0004	0.0036	0.0347	0.2629 45	0.9693 89	1.9252 99	2.9205	3.9201
93	0.0004	0.0037	0.0355	0.2674 46	0.9782 89	1.9351 99	2.9305	3.9301
94	0.0004	0.0038	0.0363	0.2721 47	0.9872 90	1.9450 99	2.9405	3.9400
95	0.0004	0.0039	0.0371	0.2767 47	0.9962 90	1.9548 99	2.9505	3.9500
96	0.0004	0.0039	0.0379	0.2815 48	1.0052 90	1.9647 99	2.9605	3.9600
97	0.0004	0.0040	0.0387	0.2863 48	1.0142 90	1.9746 99	2.9705	3.9700
98	0.0004	0.0041	0.0396	0.2911 49	1.0232 91	1.9845 99	2.9805	3.9800
99	0.0004	0.0042	0.0405	0.2961 49	1.0323 91	1.9944 99	2.9904	3.9900
00	0.0004	0.0043	0.0414	0.3010 50	1.0414 91	2.0043 99	3.0004	4.0000

ϕ	0° lsin	ltn	lsc	1° lsin	ltn	lsc	2° lsin	ltn	lsc	
00′	− ∞		00	8.2419 19 72		01	8.5428 31 36		03	60′
01′	6.4637 37		00	8.2490 91 71		01	8.5464 67 36		03	59′
02′	6.7648 48		00	8.2561 62 70		01	8.5500 03 36		03	58′
03′	6.9408 08		00	8.2630 31 69		01	8.5535 38 35		03	57′
04′	7.0658 58		00	8.2699*00 68		01	8.5571 73 35		03	56′
05′	7.1627 27		00	8.2766 67 67		01	8.5605 08 35		03	55′
06′	7.2419 19		00	8.2832 33 66		01	8.5640 43 34		03	54′
07′	7.3088 88		00	8.2898 99 65		01	8.5674 77 34		03	53′
08′	7.3668 68		00	8.2962 63 64		01	8.5708 11 34		03	52′
09′	7.4180 80		00	8.3025 26 63		01	8.5742 45 34		03	51′
10′	7.4637 37		00	8.3088 89 62		01	8.5776 79 33		03	50′
11′	7.5051 51		00	8.3150 50 61		01	8.5809 12 33		03	49′
12′	7.5429 29		00	8.3210 11 60		01	8.5842 45 33		03	48′
13′	7.5777 77		00	8.3270 71 59		01	8.5875 78 33		03	47′
14′	7.6099 99		00	8.3329 30 59		01	8.5907 11 32		03	46′
15′	7.6398 98		00	8.3388 89 58		01	8.5939 43 32		03	45′
16′	7.6678 78		00	8.3445 46 57		01	8.5972 75 32		03	44′
17′	7.6942 42		00	8.3502 03 56		01	8.6003 07 32		03	43′
18′	7.7190 90		00	8.3558 59 56		01	8.6035 38 31		04	42′
19′	7.7425 25		00	8.3613 14 55		01	8.6066 70 31		04	41′
20′	7.7648 48		00	8.3668 69 54		01	8.6097*01 31		04	40′
21′	7.7859 60		00	8.3722 23 54		01	8.6128 32 31		04	39′
22′	7.8061 62		00	8.3775 76 53		01	8.6159 63 31		04	38′
23′	7.8255 55		00	8.3828 29 52		01	8.6189 93 30		04	37′
24′	7.8439 39		00	8.3880 81 52		01	8.6220 23 30		04	36′
25′	7.8617 17		00	8.3931 32 51		01	8.6250 54 30		04	35′
26′	7.8787 87		00	8.3982 83 51		01	8.6279 83 30		04	34′
27′	7.8951 51		00	8.4032 33 50		01	8.6309 13 30		04	33′
28′	7.9109 09		00	8.4082 83 49		01	8.6339 43 29		04	32′
29′	7.9261 61		00	8.4131 32 49		01	8.6368 72 29		04	31′
30′	7.9408 09		00	8.4179 81 48		01	8.6397*01 29		04	30′
31′	7.9551 51		00	8.4227 29 48		02	8.6426 30 29		04	29′
32′	7.9689 89		00	8.4275 76 47		02	8.6454 59 29		04	28′
33′	7.9822 23	132	00	8.4322 23 47		02	8.6483 87 28		04	27′
34′	7.9952 52	128	00	8.4368 70 46		02	8.6511 15 28		04	26′
35′	8.0078 78	124	00	8.4414 16 46		02	8.6539 44 28		04	25′
36′	8.0200 00	121	00	8.4459 61 45		02	8.6567 71 28		04	24′
37′	8.0319 19	117	00	8.4504 06 45		02	8.6595 99 28		05	23′
38′	8.0435 35	114	00	8.4549 51 44		02	8.6622 27 27		05	22′
39′	8.0548 48	111	00	8.4593 95 44		02	8.6650 54 27		05	21′
40′	8.0658 58	109	00	8.4637 38 43		02	8.6677 82 27		05	20′
41′	8.0765 65	106	00	8.4680 82 43		02	8.6704 09 27		05	19′
42′	8.0870 70	103	00	8.4723 25 43		02	8.6731 36 27		05	18′
43′	8.0972 72	101	00	8.4765 67 42		02	8.6758 62 27		05	17′
44′	8.1072 72	99	00	8.4807 09 42		02	8.6784 89 26		05	16′
45′	8.1169 70	97	00	8.4848 51 41		02	8.6810 15 26		05	15′
46′	8.1265 65	94	00	8.4890 92 41		02	8.6837 42 26		05	14′
47′	8.1358 59	92	00	8.4930 33 41		02	8.6863 68 26		05	13′
48′	8.1450 50	90	00	8.4971 73 40		02	8.6889 94 26		05	12′
49′	8.1539 40	89	00	8.5011 13 40		02	8.6914 20 26		05	11′
50′	8.1627 27	87	00	8.5050 53 39		02	8.6940 45 26		05	10′
51′	8.1713 13	85	00	8.5090 92 39		02	8.6965 71 25		05	09′
52′	8.1797 98	84	00	8.5129 31 39		02	8.6991 96 25		05	08′
53′	8.1880 80	82	01	8.5167 70 38		02	8.7016 21 25		06	07′
54′	8.1961 62	80	01	8.5206 08 38		02	8.7041 46 25		06	06′
55′	8.2041 41	79	01	8.5243 46 38		02	8.7066 71 25		06	05′
56′	8.2119 20	78	01	8.5281 83 37		02	8.7090 96 25		06	04′
57′	8.2196 96	76	01	8.5318 21 37		03	8.7115 21 25		06	03′
58′	8.2271 72	75	01	8.5355 58 37		03	8.7140 45 24		06	02′
59′	8.2346 46	74	01	8.5392 94 37		03	8.7164 70 24		06	01′
60′	8.2419 19	72	01	8.5428 31 36		03	8.7188 94 24		06	00′
	89° lcos	ltn	lcsc	88° lcos	ltn	lcsc	87° lcos	ltn	lcsc	θ

ϕ	3° lsin	ltn	lsc	4° lsin	ltn	lsc	5° lsin	ltn	lsc	
00′	8.7188 94	24	06	8.8436 46	18	11	8.9403 20	14	17	60′
01′	8.7212 18	24	06	8.8454 65	18	11	8.9417 34	14	17	59′
02′	8.7236 42	24	06	8.8472 83	18	11	8.9432 49	14	17	58′
03′	8.7260 66	24	06	8.8490*01	18	11	8.9446 63	14	17	57′
04′	8.7283 90	24	06	8.8508 18	18	11	8.9460 77	14	17	56′
05′	8.7307 13	23	06	8.8525 36	18	11	8.9475 92	14	17	55′
06′	8.7330 37	23	06	8.8543 54	18	11	8.9489*06	14	17	54′
07′	8.7354 60	23	06	8.8560 72	18	11	8.9503 20	14	17	53′
08′	8.7377 83	23	06	8.8578 89	18	11	8.9517 34	14	17	52′
09′	8.7400 06	23	07	8.8595*07	17	11	8.9531 49	14	18	51′
10′	8.7423 29	23	07	8.8613 24	17	11	8.9545 63	14	18	50′
11′	8.7445 52	23	07	8.8630 42	17	12	8.9559 77	14	18	49′
12′	8.7468 75	23	07	8.8647 59	17	12	8.9573 91	14	18	48′
13′	8.7491 97	23	07	8.8665 76	17	12	8.9587*05	14	18	47′
14′	8.7513 20	22	07	8.8682 94	17	12	8.9601 19	14	18	46′
15′	8.7535 42	22	07	8.8699*11	17	12	8.9614 33	14	18	45′
16′	8.7557 65	22	07	8.8716 28	17	12	8.9628 46	14	18	44′
17′	8.7580 87	22	07	8.8733 45	17	12	8.9642 60	14	18	43′
18′	8.7602 09	22	07	8.8749 62	17	12	8.9655 74	14	19	42′
19′	8.7623 31	22	07	8.8766 78	17	12	8.9669 88	14	19	41′
20′	8.7645 52	22	07	8.8783 95	17	12	8.9682*01	14	19	40′
21′	8.7667 74	22	07	8.8799*12	17	13	8.9696*15	14	19	39′
22′	8.7688 96	22	08	8.8816 29	17	13	8.9709 29	14	19	38′
23′	8.7710 17	21	08	8.8833 45	17	13	8.9723 42	13	19	37′
24′	8.7731 39	21	08	8.8849 62	16	13	8.9736 56	13	19	36′
25′	8.7752 60	21	08	8.8865 78	16	13	8.9750 69	13	19	35′
26′	8.7773 81	21	08	8.8882 95	16	13	8.9763 82	13	20	34′
27′	8.7794*02	21	08	8.8898*11	16	13	8.9776 96	13	20	33′
28′	8.7815 23	21	08	8.8914 27	16	13	8.9789*09	13	20	32′
29′	8.7836 44	21	08	8.8930 44	16	13	8.9803 23	13	20	31′
30′	8.7857 65	21	08	8.8946 60	16	13	8.9816 36	13	20	30′
31′	8.7877 86	21	08	8.8962 76	16	14	8.9829 49	13	20	29′
32′	8.7898*06	20	08	8.8978 92	16	14	8.9842 62	13	20	28′
33′	8.7918 27	20	08	8.8994*08	16	14	8.9855 75	13	20	27′
34′	8.7939 47	20	08	8.9010 24	16	14	8.9868 88	13	21	26′
35′	8.7959 67	20	08	8.9026 40	16	14	8.9881*01	13	21	25′
36′	8.7979 88	20	09	8.9042 56	16	14	8.9894*15	13	21	24′
37′	8.7999*08	20	09	8.9057 71	16	14	8.9907 28	13	21	23′
38′	8.8019 28	20	09	8.9073 87	16	14	8.9919 40	13	21	22′
39′	8.8039 48	20	09	8.9089*03	16	14	8.9932 53	13	21	21′
40′	8.8059 67	20	09	8.9104 18	16	14	8.9945 66	13	21	20′
41′	8.8078 87	20	09	8.9119 34	15	15	8.9958 79	13	21	19′
42′	8.8098*07	20	09	8.9135 50	15	15	8.9970 92	13	22	18′
43′	8.8117 26	19	09	8.9150 65	15	15	8.9983*05	13	22	17′
44′	8.8137 46	19	09	8.9166 80	15	15	8.9996*17	13	22	16′
45′	8.8156 65	19	09	8.9181 96	15	15	9.0008 30	13	22	15′
46′	8.8175 85	19	09	8.9196*11	15	15	9.0021 43	13	22	14′
47′	8.8194*04	19	09	8.9211 26	15	15	9.0033 55	13	22	13′
48′	8.8213 23	19	10	8.9226 41	15	15	9.0046 68	12	22	12′
49′	8.8232 42	19	10	8.9241 56	15	15	9.0058 80	12	22	11′
50′	8.8251 61	19	10	8.9256 72	15	15	9.0070 93	12	23	10′
51′	8.8270 80	19	10	8.9271 87	15	16	9.0083*05	12	23	09′
52′	8.8289 99	19	10	8.9286*02	15	16	9.0095*18	12	23	08′
53′	8.8307 17	19	10	8.9301 16	15	16	9.0107 30	12	23	07′
54′	8.8326 36	19	10	8.9315 31	15	16	9.0120 43	12	23	06′
55′	8.8345 55	19	10	8.9330 46	15	16	9.0132 55	12	23	05′
56′	8.8363 73	18	10	8.9345 61	15	16	9.0144 67	12	23	04′
57′	8.8381 92	18	10	8.9359 76	15	16	9.0156 80	12	23	03′
58′	8.8400 10	18	10	8.9374 90	15	16	9.0168 92	12	24	02′
59′	8.8418 28	18	11	8.9388*05	15	16	9.0180*04	12	24	01′
60′	8.8436 46	18	11	8.9403 20	14	17	9.0192*16	12	24	00′
	86° lcos	ltn	lcsc	85° lcos	ltn	lcsc	84° lcos	ltn	lcsc	θ

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ϕ	S	$l \sin \phi$	ϕ	T	$l \tan \phi$	ϕ	$l \sec \phi$	ϕ	$l \sec \phi$
	6.46			**6.46**			**0.00**		**0.00**
0°00′.000		$-\infty$	0°00′.000		$-\infty$	0°00′.000	00	4°20′.701	13
	37		0°44′.155	37	8.1087				
1°51′.479		8.5108		38	8.4663	0°52′.164	01	4°30′.918	14
	36		1°40′.555			1°30′.348		4°40′.762	
2°49′.567		8.6929	2°15′.168	39	8.5948	1°56′.634	02	4°50′.271	15
	35		2°42′.563	40	8.6751				
3°32′.313		8.7904	3°05′.959	41	8.7336	2°17′.998	03	4°59′.477	16
	34		3°26′.717	42	8.7796	2°36′.469	04	5°08′.407	17
4°07′.789		8.8574	3°45′.567	43	8.8176	2°52′.976	05	5°17′.084	18
	33		4°02′.954	44	8.8500	3°08′.038	06	5°25′.528	19
4°38′.783		8.9085	4°19′.171	45	8.8781	3°21′.977	07	5°33′.758	20
	32		4°34′.427	46	8.9031	3°35′.016	08	5°41′.789	21
5°06′.659		8.9498	4°48′.875	47	8.9255	3°47′.300	09	5°49′.633	22
	31		5°02′.628	48	8.9458	3°58′.955	10	5°57′.305	23
5°32′.201		8.9845	5°15′.780	49	8.9643	4°10′.064	11	6°04′.814	24
	30		5°28′.401	50	8.9815	4°20′.701	12	6°12′.170	25
5°55′.913		9.0143	5°40′.550	51	8.9973				
	29		5°52′.278	52	9.0121				
6°18′.138		9.0405	6°03′.623	53	9.0260				

ϕ	$l \sin \phi$		$l \csc \phi$	$l \tan \phi$		$l \operatorname{ctn} \phi$	$l \sec \phi$	$l \cos \phi$	
6° 00′	9.0192	120	0.9808	9.0216	122	0.9784	0.0024 1	9.9976	**84° 00′**
6° 10′	9.0311	117	0.9689	9.0336	118	0.9664	0.0025 1	9.9975	83° 50′
6° 20′	9.0426	114	0.9574	9.0453	115	0.9547	0.0027 1	9.9973	83° 40′
6° 30′	9.0539	111	0.9461	9.0567	112	0.9433	0.0028 1	9.9972	83° 30′
6° 40′	9.0648	108	0.9352	9.0678	110	0.9322	0.0029 1	9.9971	83° 20′
6° 50′	9.0755	105	0.9245	9.0786	107	0.9214	0.0031 2	9.9969	83° 10′
7° 00′	9.0859	103	0.9141	9.0891	104	0.9109	0.0032 2	9.9968	**83° 00′**
7° 10′	9.0961	100	0.9039	9.0995	102	0.9005	0.0034 2	9.9966	82° 50′
7° 20′	9.1060	98	0.8940	9.1096	100	0.8904	0.0036 2	9.9964	82° 40′
7° 30′	9.1157	96	0.8843	9.1194	98	0.8806	0.0037 2	9.9963	82° 30′
7° 40′	9.1252	94	0.8748	9.1291	96	0.8709	0.0039 2	9.9961	82° 20′
7° 50′	9.1345	92	0.8655	9.1385	94	0.8615	0.0041 2	9.9959	82° 10′
8° 00′	9.1436	90	0.8564	9.1478	92	0.8522	0.0042 2	9.9958	**82° 00′**
8° 10′	9.1525	88	0.8475	9.1569	90	0.8431	0.0044 2	9.9956	81° 50′
8° 20′	9.1612	86	0.8388	9.1658	88	0.8342	0.0046 2	9.9954	81° 40′
8° 30′	9.1697	85	0.8303	9.1745	86	0.8255	0.0048 2	9.9952	81° 30′
8° 40′	9.1781	83	0.8219	9.1831	85	0.8169	0.0050 2	9.9950	81° 20′
8° 50′	9.1863	81	0.8137	9.1915	83	0.8085	0.0052 2	9.9948	81° 10′
9° 00′	9.1943	80	0.8057	9.1997	82	0.8003	0.0054 2	9.9946	**81° 00′**
9° 10′	9.2022	78	0.7978	9.2078	80	0.7922	0.0056 2	9.9944	80° 50′
9° 20′	9.2100	77	0.7900	9.2158	79	0.7842	0.0058 2	9.9942	80° 40′
9° 30′	9.2176	75	0.7824	9.2236	78	0.7764	0.0060 2	9.9940	80° 30′
9° 40′	9.2251	74	0.7749	9.2313	76	0.7687	0.0062 2	9.9938	80° 20′
9° 50′	9.2324	73	0.7676	9.2389	75	0.7611	0.0064 2	9.9936	80° 10′
10° 00′	9.2397	72	0.7603	9.2463	74	0.7537	0.0066 2	9.9934	**80° 00′**
	$l \cos \theta$		$l \sec \theta$	$l \operatorname{ctn} \theta$		$l \tan \theta$	$l \csc \theta$	$l \sin \theta$	θ

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ϕ	$l\sin\phi$	$l\csc\phi$	$l\tan\phi$	$l\operatorname{ctn}\phi$	$l\sec\phi$	$l\cos\phi$	
10° 00′	9.2397 72	0.7603	9.2463 74	0.7537	0.0066 2	9.9934	**80° 00′**
10° 10′	9.2468 70	0.7532	9.2536 73	0.7464	0.0069 2	9.9931	79° 50′
10° 20′	9.2538 69	0.7462	9.2609 72	0.7391	0.0071 2	9.9929	79° 40′
10° 30′	9.2606 68	0.7394	9.2680 71	0.7320	0.0073 2	9.9927	79° 30′
10° 40′	9.2674 67	0.7326	9.2750 69	0.7250	0.0076 2	9.9924	79° 20′
10° 50′	9.2740 66	0.7260	9.2819 68	0.7181	0.0078 2	9.9922	79° 10′
11° 00′	9.2806 65	0.7194	9.2887 67	0.7113	0.0081 2	9.9919	**79° 00′**
11° 10′	9.2870 64	0.7130	9.2953 66	0.7047	0.0083 2	9.9917	78° 50′
11° 20′	9.2934 63	0.7066	9.3020 66	0.6980	0.0086 3	9.9914	78° 40′
11° 30′	9.2997 62	0.7003	9.3085 65	0.6915	0.0088 3	9.9912	78° 30′
11° 40′	9.3058 61	0.6942	9.3149 64	0.6851	0.0091 3	9.9909	78° 20′
11° 50′	9.3119 60	0.6881	9.3212 63	0.6788	0.0093 3	9.9907	78° 10′
12° 00′	9.3179 59	0.6821	9.3275 62	0.6725	0.0096 3	9.9904	**78° 00′**
12° 10′	9.3238 59	0.6762	9.3336 61	0.6664	0.0099 3	9.9901	77° 50′
12° 20′	9.3296 58	0.6704	9.3397 61	0.6603	0.0101 3	9.9899	77° 40′
12° 30′	9.3353 57	0.6647	9.3458 60	0.6542	0.0104 3	9.9896	77° 30′
12° 40′	9.3410 56	0.6590	9.3517 59	0.6483	0.0107 3	9.9893	77° 20′
12° 50′	9.3466 55	0.6534	9.3576 58	0.6424	0.0110 3	9.9890	77° 10′
13° 00′	9.3521 55	0.6479	9.3634 58	0.6366	0.0113 3	9.9887	**77° 00′**
13° 10′	9.3575 54	0.6425	9.3691 57	0.6309	0.0116 3	9.9884	76° 50′
13° 20′	9.3629 53	0.6371	9.3748 56	0.6252	0.0119 3	9.9881	76° 40′
13° 30′	9.3682 53	0.6318	9.3804 56	0.6196	0.0122 3	9.9878	76° 30′
13° 40′	9.3734 52	0.6266	9.3859 55	0.6141	0.0125 3	9.9875	76° 20′
13° 50′	9.3786 51	0.6214	9.3914 54	0.6086	0.0128 3	9.9872	76° 10′
14° 00′	9.3837 51	0.6163	9.3968 54	0.6032	0.0131 3	9.9869	**76° 00′**
14° 10′	9.3887 50	0.6113	9.4021 53	0.5979	0.0134 3	9.9866	75° 50′
14° 20′	9.3937 49	0.6063	9.4074 53	0.5926	0.0137 3	9.9863	75° 40′
14° 30′	9.3986 49	0.6014	9.4127 52	0.5873	0.0141 3	9.9859	75° 30′
14° 40′	9.4035 48	0.5965	9.4178 52	0.5822	0.0144 3	9.9856	75° 20′
14° 50′	9.4083 48	0.5917	9.4230 51	0.5770	0.0147 3	9.9853	75° 10′
15° 00′	9.4130 47	0.5870	9.4281 51	0.5719	0.0151 3	9.9849	**75° 00′**
15° 10′	9.4177 47	0.5823	9.4331 50	0.5669	0.0154 3	9.9846	74° 50′
15° 20′	9.4223 46	0.5777	9.4381 50	0.5619	0.0157 3	9.9843	74° 40′
15° 30′	9.4269 46	0.5731	9.4430 49	0.5570	0.0161 4	9.9839	74° 30′
15° 40′	9.4314 45	0.5686	9.4479 49	0.5521	0.0164 4	9.9836	74° 20′
15° 50′	9.4359 45	0.5641	9.4527 48	0.5473	0.0168 4	9.9832	74° 10′
16° 00′	9.4403 44	0.5597	9.4575 48	0.5425	0.0172 4	9.9828	**74° 00′**
16° 10′	9.4447 44	0.5553	9.4622 47	0.5378	0.0175 4	9.9825	73° 50′
16° 20′	9.4491 43	0.5509	9.4669 47	0.5331	0.0179 4	9.9821	73° 40′
16° 30′	9.4533 43	0.5467	9.4716 46	0.5284	0.0183 4	9.9817	73° 30′
16° 40′	9.4576 42	0.5424	9.4762 46	0.5238	0.0186 4	9.9814	73° 20′
16° 50′	9.4618 42	0.5382	9.4808 46	0.5192	0.0190 4	9.9810	73° 10′
17° 00′	9.4659 41	0.5341	9.4853 45	0.5147	0.0194 4	9.9806	**73° 00′**
	$l\cos\theta$	$l\sec\theta$	$l\operatorname{ctn}\theta$	$l\tan\theta$	$l\csc\theta$	$l\sin\theta$	θ

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ϕ	l sin ϕ	l csc ϕ	l tan ϕ	l ctn ϕ	l sec ϕ	l cos ϕ	
17° 00′	9.4659 41	0.5341	9.4853 45	0.5147	0.0194 4	9.9806	**73° 00′**
17° 10′	9.4700 41	0.5300	9.4898 45	0.5102	0.0198 4	9.9802	72° 50′
17° 20′	9.4741 40	0.5259	9.4943 44	0.5057	0.0202 4	9.9798	72° 40′
17° 30′	9.4781 40	0.5219	9.4987 44	0.5013	0.0206 4	9.9794	72° 30′
17° 40′	9.4821 40	0.5179	9.5031 44	0.4969	0.0210 4	9.9790	72° 20′
17° 50′	9.4861 39	0.5139	9.5075 43	0.4925	0.0214 4	9.9786	72° 10′
18° 00′	9.4900 39	0.5100	9.5118 43	0.4882	0.0218 4	9.9782	**72° 00′**
18° 10′	9.4939 38	0.5061	9.5161 43	0.4839	0.0222 4	9.9778	71° 50′
18° 20′	9.4977 38	0.5023	9.5203 42	0.4797	0.0226 4	9.9774	71° 40′
18° 30′	9.5015 38	0.4985	9.5245 42	0.4755	0.0230 4	9.9770	71° 30′
18° 40′	9.5052 37	0.4948	9.5287 42	0.4713	0.0235 4	9.9765	71° 20′
18° 50′	9.5090 37	0.4910	9.5329 41	0.4671	0.0239 4	9.9761	71° 10′
19° 00′	9.5126 37	0.4874	9.5370 41	0.4630	0.0243 4	9.9757	**71° 00′**
19° 10′	9.5163 36	0.4837	9.5411 41	0.4589	0.0248 4	9.9752	70° 50′
19° 20′	9.5199 36	0.4801	9.5451 40	0.4549	0.0252 4	9.9748	70° 40′
19° 30′	9.5235 36	0.4765	9.5491 40	0.4509	0.0257 4	9.9743	70° 30′
19° 40′	9.5270 35	0.4730	9.5531 40	0.4469	0.0261 5	9.9739	70° 20′
19° 50′	9.5306 35	0.4694	9.5571 40	0.4429	0.0266 5	9.9734	70° 10′
20° 00′	9.5341 35	0.4659	9.5611 39	0.4389	0.0270 5	9.9730	**70° 00′**
20° 10′	9.5375 34	0.4625	9.5650 39	0.4350	0.0275 5	9.9725	69° 50′
20° 20′	9.5409 34	0.4591	9.5689 39	0.4311	0.0279 5	9.9721	69° 40′
20° 30′	9.5443 34	0.4557	9.5727 39	0.4273	0.0284 5	9.9716	69° 30′
20° 40′	9.5477 33	0.4523	9.5766 38	0.4234	0.0289 5	9.9711	69° 20′
20° 50′	9.5510 33	0.4490	9.5804 38	0.4196	0.0294 5	9.9706	69° 10′
21° 00′	9.5543 33	0.4457	9.5842 38	0.4158	0.0298 5	9.9702	**69° 00′**
21° 10′	9.5576 33	0.4424	9.5879 38	0.4121	0.0303 5	9.9697	68° 50′
21° 20′	9.5609 32	0.4391	9.5917 37	0.4083	0.0308 5	9.9692	68° 40′
21° 30′	9.5641 32	0.4359	9.5954 37	0.4046	0.0313 5	9.9687	68° 30′
21° 40′	9.5673 32	0.4327	9.5991 37	0.4009	0.0318 5	9.9682	68° 20′
21° 50′	9.5704 32	0.4296	9.6028 37	0.3972	0.0323 5	9.9677	68° 10′
22° 00′	9.5736 31	0.4264	9.6064 36	0.3936	0.0328 5	9.9672	**68° 00′**
22° 10′	9.5767 31	0.4233	9.6100 36	0.3900	0.0333 5	9.9667	67° 50′
22° 20′	9.5798 31	0.4202	9.6136 36	0.3864	0.0339 5	9.9661	67° 40′
22° 30′	9.5828 30	0.4172	9.6172 36	0.3828	0.0344 5	9.9656	67 30′
22° 40′	9.5859 30	0.4141	9.6208 36	0.3792	0.0349 5	9.9651	67° 20′
22° 50′	9.5889 30	0.4111	9.6243 35	0.3757	0.0354 5	9.9646	67° 10′
23° 00′	9.5919 30	0.4081	9.6279 35	0.3721	0.0360 5	9.9640	**67° 00′**
23° 10′	9.5948 30	0.4052	9.6314 35	0.3686	0.0365 5	9.9635	66° 50′
23° 20′	9.5978 29	0.4022	9.6348 35	0.3652	0.0371 5	9.9629	66° 40′
23° 30′	9.6007 29	0.3993	9.6383 35	0.3617	0.0376 5	9.9624	66° 30′
23° 40′	9.6036 29	0.3964	9.6417 34	0.3583	0.0382 6	9.9618	66° 20′
23° 50′	9.6065 29	0.3935	9.6452 34	0.3548	0.0387 6	9.9613	66° 10′
24° 00′	9.6093 28	0.3907	9.6486 34	0.3514	0.0393 6	9.9607	**66° 00′**
	l cos θ	l sec θ	l ctn θ	l tan θ	l csc θ	l sin θ	θ

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ϕ	$l\sin\phi$	$l\csc\phi$	$l\tan\phi$	$l\operatorname{ctn}\phi$	$l\sec\phi$	$l\cos\phi$	
24° 00′	9.6093 28	0.3907	9.6486 34	0.3514	0.0393 6	9.9607	**66° 00′**
24° 10′	9.6121 28	0.3879	9.6520 34	0.3480	0.0398 6	9.9602	65° 50′
24° 20′	9.6149 28	0.3851	9.6553 34	0.3447	0.0404 6	9.9596	65° 40′
24° 30′	9.6177 28	0.3823	9.6587 33	0.3413	0.0410 6	9.9590	65° 30′
24° 40′	9.6205 28	0.3795	9.6620 33	0.3380	0.0416 6	9.9584	65° 20′
24° 50′	9.6232 27	0.3768	9.6654 33	0.3346	0.0421 6	9.9579	65° 10′
25° 00′	9.6259 27	0.3741	9.6687 33	0.3313	0.0427 6	9.9573	**65° 00′**
25° 10′	9.6286 27	0.3714	9.6720 33	0.3280	0.0433 6	9.9567	64° 50′
25° 20′	9.6313 27	0.3687	9.6752 33	0.3248	0.0439 6	9.9561	64° 40′
25° 30′	9.6340 26	0.3660	9.6785 33	0.3215	0.0445 6	9.9555	64° 30′
25° 40′	9.6366 26	0.3634	9.6817 32	0.3183	0.0451 6	9.9549	64° 20′
25° 50′	9.6392 26	0.3608	9.6850 32	0.3150	0.0457 6	9.9543	64° 10′
26° 00′	9.6418 26	0.3582	9.6882 32	0.3118	0.0463 6	9.9537	**64° 00′**
26° 10′	9.6444 26	0.3556	9.6914 32	0.3086	0.0470 6	9.9530	63° 50′
26° 20′	9.6470 26	0.3530	9.6946 32	0.3054	0.0476 6	9.9524	63° 40′
26° 30′	9.6495 25	0.3505	9.6977 32	0.3023	0.0482 6	9.9518	63° 30′
26° 40′	9.6521 25	0.3479	9.7009 31	0.2991	0.0488 6	9.9512	63° 20′
26° 50′	9.6546 25	0.3454	9.7040 31	0.2960	0.0495 6	9.9505	63° 10′
27° 00′	9.6570 25	0.3430	9.7072 31	0.2928	0.0501 6	9.9499	**63° 00′**
27° 10′	9.6595 25	0.3405	9.7103 31	0.2897	0.0508 6	9.9492	62° 50′
27° 20′	9.6620 24	0.3380	9.7134 31	0.2866	0.0514 7	9.9486	62° 40′
27° 30′	9.6644 24	0.3356	9.7165 31	0.2835	0.0521 7	9.9479	62° 30′
27° 40′	9.6668 24	0.3332	9.7196 31	0.2804	0.0527 7	9.9473	62° 20′
27° 50′	9.6692 24	0.3308	9.7226 31	0.2774	0.0534 7	9.9466	62° 10′
28° 00′	9.6716 24	0.3284	9.7257 30	0.2743	0.0541 7	9.9459	**62° 00′**
28° 10′	9.6740 24	0.3260	9.7287 30	0.2713	0.0547 7	9.9453	61° 50′
28° 20′	9.6763 23	0.3237	9.7317 30	0.2683	0.0554 7	9.9446	61° 40′
28° 30′	9.6787 23	0.3213	9.7348 30	0.2652	0.0561 7	9.9439	61° 30′
28° 40′	9.6810 23	0.3190	9.7378 30	0.2622	0.0568 7	9.9432	61° 20′
28° 50′	9.6833 23	0.3167	9.7408 30	0.2592	0.0575 7	9.9425	61° 10′
29° 00′	9.6856 23	0.3144	9.7438 30	0.2562	0.0582 7	9.9418	**61° 00′**
29° 10′	9.6878 23	0.3122	9.7467 30	0.2533	0.0589 7	9.9411	60° 50′
29° 20′	9.6901 22	0.3099	9.7497 30	0.2503	0.0596 7	9.9404	60° 40′
29° 30′	9.6923 22	0.3077	9.7526 29	0.2474	0.0603 7	9.9397	60° 30′
29° 40′	9.6946 22	0.3054	9.7556 29	0.2444	0.0610 7	9.9390	60° 20′
29° 50′	9.6968 22	0.3032	9.7585 29	0.2415	0.0617 7	9.9383	60° 10′
30° 00′	9.6990 22	0.3010	9.7614 29	0.2386	0.0625 7	9.9375	**60° 00′**
30° 10′	9.7012 22	0.2988	9.7644 29	0.2356	0.0632 7	9.9368	59° 50′
30° 20′	9.7033 22	0.2967	9.7673 29	0.2327	0.0639 7	9.9361	59° 40′
30° 30′	9.7055 21	0.2945	9.7701 29	0.2299	0.0647 7	9.9353	59° 30′
30° 40′	9.7076 21	0.2924	9.7730 29	0.2270	0.0654 7	9.9346	59° 20′
30° 50′	9.7097 21	0.2903	9.7759 29	0.2241	0.0662 8	9.9338	59° 10′
31° 00′	9.7118 21	0.2882	9.7788 29	0.2212	0.0669 8	9.9331	**59° 00′**
	$l\cos\theta$	$l\sec\theta$	$l\operatorname{ctn}\theta$	$l\tan\theta$	$l\csc\theta$	$l\sin\theta$	θ

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ϕ	l sin ϕ	l csc ϕ	l tan ϕ	l ctn ϕ	l sec ϕ	l cos ϕ	
31° 00′	9.7118 21	0.2882	9.7788 29	0.2212	0.0669 8	9.9331	**59° 00′**
31° 10′	9.7139 21	0.2861	9.7816 29	0.2184	0.0677 8	9.9323	58° 50′
31° 20′	9.7160 21	0.2840	9.7845 28	0.2155	0.0685 8	9.9315	58° 40′
31° 30′	9.7181 21	0.2819	9.7873 28	0.2127	0.0692 8	9.9308	58° 30′
31° 40′	9.7201 20	0.2799	9.7902 28	0.2098	0.0700 8	9.9300	58° 20′
31° 50′	9.7222 20	0.2778	9.7930 28	0.2070	0.0708 8	9.9292	58° 10′
32° 00′	9.7242 20	0.2758	9.7958 28	0.2042	0.0716 8	9.9284	**58° 00′**
32° 10′	9.7262 20	0.2738	9.7986 28	0.2014	0.0724 8	9.9276	57° 50′
32° 20′	9.7282 20	0.2718	9.8014 28	0.1986	0.0732 8	9.9268	57° 40′
32° 30′	9.7302 20	0.2698	9.8042 28	0.1958	0.0740 8	9.9260	57° 30′
32° 40′	9.7322 20	0.2678	9.8070 28	0.1930	0.0748 8	9.9252	57° 20′
32° 50′	9.7342 20	0.2658	9.8097 28	0.1903	0.0756 8	9.9244	57° 10′
33° 00′	9.7361 19	0.2639	9.8125 28	0.1875	0.0764 8	9.9236	**57° 00′**
33° 10′	9.7380 19	0.2620	9.8153 28	0.1847	0.0772 8	9.9228	56° 50′
33° 20′	9.7400 19	0.2600	9.8180 28	0.1820	0.0781 8	9.9219	56° 40′
33° 30′	9.7419 19	0.2581	9.8208 27	0.1792	0.0789 8	9.9211	56° 30′
33° 40′	9.7438 19	0.2562	9.8235 27	0.1765	0.0797 8	9.9203	56° 20′
33° 50′	9.7457 19	0.2543	9.8263 27	0.1737	0.0806 8	9.9194	56° 10′
34° 00′	9.7476 19	0.2524	9.8290 27	0.1710	0.0814 9	9.9186	**56° 00′**
34° 10′	9.7494 19	0.2506	9.8317 27	0.1683	0.0823 9	9.9177	55° 50′
34° 20′	9.7513 18	0.2487	9.8344 27	0.1656	0.0831 9	9.9169	55° 40′
34° 30′	9.7531 18	0.2469	9.8371 27	0.1629	0.0840 9	9.9160	55° 30′
34° 40′	9.7550 18	0.2450	9.8398 27	0.1602	0.0849 9	9.9151	55° 20′
34° 50′	9.7568 18	0.2432	9.8425 27	0.1575	0.0858 9	9.9142	55° 10′
35° 00′	9.7586 18	0.2414	9.8452 27	0.1548	0.0866 9	9.9134	**55° 00′**
35° 10′	9.7604 18	0.2396	9.8479 27	0.1521	0.0875 9	9.9125	54° 50′
35° 20′	9.7622 18	0.2378	9.8506 27	0.1494	0.0884 9	9.9116	54° 40′
35° 30′	9.7640 18	0.2360	9.8533 27	0.1467	0.0893 9	9.9107	54° 30′
35° 40′	9.7657 18	0.2343	9.8559 27	0.1441	0.0902 9	9.9098	54° 20′
35° 50′	9.7675 17	0.2325	9.8586 27	0.1414	0.0911 9	9.9089	54° 10′
36° 00′	9.7692 17	0.2308	9.8613 27	0.1387	0.0920 9	9.9080	**54° 00′**
36° 10′	9.7710 17	0.2290	9.8639 27	0.1361	0.0930 9	9.9070	53° 50′
36° 20′	9.7727 17	0.2273	9.8666 26	0.1334	0.0939 9	9.9061	53° 40′
36° 30′	9.7744 17	0.2256	9.8692 26	0.1308	0.0948 9	9.9052	53° 30′
36° 40′	9.7761 17	0.2239	9.8718 26	0.1282	0.0958 9	9.9042	53° 20′
36° 50′	9.7778 17	0.2222	9.8745 26	0.1255	0.0967 9	9.9033	53° 10′
37° 00′	9.7795 17	0.2205	9.8771 26	0.1229	0.0977 10	9.9023	**53° 00′**
37° 10′	9.7811 17	0.2189	9.8797 26	0.1203	0.0986 10	9.9014	52° 50′
37° 20′	9.7828 17	0.2172	9.8824 26	0.1176	0.0996 10	9.9004	52° 40′
37° 30′	9.7844 16	0.2156	9.8850 26	0.1150	0.1005 10	9.8995	52° 30′
37° 40′	9.7861 16	0.2139	9.8876 26	0.1124	0.1015 10	9.8985	52° 20′
37° 50′	9.7877 16	0.2123	9.8902 26	0.1098	0.1025 10	9.8975	52° 10′
38° 00′	9.7893 16	0.2107	9.8928 26	0.1072	0.1035 10	9.8965	**52° 00′**
	l cos θ	l sec θ	l ctn θ	l tan θ	l csc θ	l sin θ	θ

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ϕ	l sin ϕ	l csc ϕ	l tan ϕ	l ctn ϕ	l sec ϕ	l cos ϕ	
38° 00′	9.7893 16	0.2107	9.8928 26	0.1072	0.1035 10	9.8965	**52° 00′**
38° 10′	9.7910 16	0.2090	9.8954 26	0.1046	0.1045 10	9.8955	51° 50′
38° 20′	9.7926 16	0.2074	9.8980 26	0.1020	0.1055 10	9.8945	51° 40′
38° 30′	9.7941 16	0.2059	9.9006 26	0.0994	0.1065 10	9.8935	51° 30′
38° 40′	9.7957 16	0.2043	9.9032 26	0.0968	0.1075 10	9.8925	51° 20′
38° 50′	9.7973 16	0.2027	9.9058 26	0.0942	0.1085 10	9.8915	51° 10′
39° 00′	9.7989 16	0.2011	9.9084 26	0.0916	0.1095 10	9.8905	**51° 00′**
39° 10′	9.8004 16	0.1996	9.9110 26	0.0890	0.1105 10	9.8895	50° 50′
39° 20′	9.8020 15	0.1980	9.9135 26	0.0865	0.1116 10	9.8884	50° 40′
39° 30′	9.8035 15	0.1965	9.9161 26	0.0839	0.1126 10	9.8874	50° 30′
39° 40′	9.8050 15	0.1950	9.9187 26	0.0813	0.1136 10	9.8864	50° 20′
39° 50′	9.8066 15	0.1934	9.9212 26	0.0788	0.1147 11	9.8853	50° 10′
40° 00′	9.8081 15	0.1919	9.9238 26	0.0762	0.1157 11	9.8843	**50° 00′**
40° 10′	9.8096 15	0.1904	9.9264 26	0.0736	0.1168 11	9.8832	49° 50′
40° 20′	9.8111 15	0.1889	9.9289 26	0.0711	0.1179 11	9.8821	49° 40′
40° 30′	9.8125 15	0.1875	9.9315 26	0.0685	0.1190 11	9.8810	49° 30′
40° 40′	9.8140 15	0.1860	9.9341 26	0.0659	0.1200 11	9.8800	49° 20′
40° 50′	9.8155 15	0.1845	9.9366 26	0.0634	0.1211 11	9.8789	49° 10′
41° 00′	9.8169 15	0.1831	9.9392 26	0.0608	0.1222 11	9.8778	**49° 00′**
41° 10′	9.8184 14	0.1816	9.9417 25	0.0583	0.1233 11	9.8767	48° 50′
41° 20′	9.8198 14	0.1802	9.9443 25	0.0557	0.1244 11	9.8756	48° 40′
41° 30′	9.8213 14	0.1787	9.9468 25	0.0532	0.1255 11	9.8745	48° 30′
41° 40′	9.8227 14	0.1773	9.9494 25	0.0506	0.1267 11	9.8733	48° 20′
41° 50′	9.8241 14	0.1759	9.9519 25	0.0481	0.1278 11	9.8722	48° 10′
42° 00′	9.8255 14	0.1745	9.9544 25	0.0456	0.1289 11	9.8711	**48° 00′**
42° 10′	9.8269 14	0.1731	9.9570 25	0.0430	0.1301 11	9.8699	47° 50′
42° 20′	9.8283 14	0.1717	9.9595 25	0.0405	0.1312 12	9.8688	47° 40′
42° 30′	9.8297 14	0.1703	9.9621 25	0.0379	0.1324 12	9.8676	47° 30′
42° 40′	9.8311 14	0.1689	9.9646 25	0.0354	0.1335 12	9.8665	47° 20′
42° 50′	9.8324 14	0.1676	9.9671 25	0.0329	0.1347 12	9.8653	47° 10′
43° 00′	9.8338 14	0.1662	9.9697 25	0.0303	0.1359 12	9.8641	**47° 00′**
43° 10′	9.8351 13	0.1649	9.9722 25	0.0278	0.1371 12	9.8629	46° 50′
43° 20′	9.8365 13	0.1635	9.9747 25	0.0253	0.1382 12	9.8618	46° 40′
43° 30′	9.8378 13	0.1622	9.9772 25	0.0228	0.1394 12	9.8606	46° 30′
43° 40′	9.8391 13	0.1609	9.9798 25	0.0202	0.1406 12	9.8594	46° 20′
43° 50′	9.8405 13	0.1595	9.9823 25	0.0177	0.1418 12	9.8582	46° 10′
44° 00′	9.8418 13	0.1582	9.9848 25	0.0152	0.1431 12	9.8569	**46° 00′**
44° 10′	9.8431 13	0.1569	9.9874 25	0.0126	0.1443 12	9.8557	45° 50′
44° 20′	9.8444 13	0.1556	9.9899 25	0.0101	0.1455 12	9.8545	45° 40′
44° 30′	9.8457 13	0.1543	9.9924 25	0.0076	0.1468 12	9.8532	45° 30′
44° 40′	9.8469 13	0.1531	9.9949 25	0.0051	0.1480 12	9.8520	45° 20′
44° 50′	9.8482 13	0.1518	9.9975 25	0.0025	0.1493 13	9.8507	45° 10′
45° 00′	9.8495 13	0.1505	0.0000 25	0.0000	0.1505 13	9.8495	**45° 00′**
	l cos θ	l sec θ	l ctn θ	l tan θ	l csc θ	l sin θ	θ

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log u	$\sin^{-1}u$	$\cos^{-1}u$	$\tan^{-1}u$	$\operatorname{ctn}^{-1}u$	log u	$\sin^{-1}u$	$\cos^{-1}u$	$\tan^{-1}u$	$\operatorname{ctn}^{-1}u$
9.	°	°	°	°	9.	°	°	°	°
00	5.74 13	84.26	5.71 13	84.29	50	18.43 44	71.57	17.55 38	72.45
01	5.87 14	84.13	5.84 13	84.16	51	18.88 45	71.12	17.93 39	72.07
02	6.01 14	83.99	5.98 14	84.02	52	19.34 46	70.66	18.32 39	71.68
03	6.15 14	83.85	6.12 14	83.88	53	19.81 48	70.19	18.72 40	71.28
04	6.30 15	83.70	6.26 14	83.74	54	20.29 49	69.71	19.12 41	70.88
05	6.44 15	83.56	6.40 15	83.60	55	20.78 50	69.22	19.54 42	70.46
06	6.59 15	83.41	6.55 15	83.45	56	21.29 51	68.71	19.95 42	70.05
07	6.75 16	83.25	6.70 15	83.30	57	21.81 53	68.19	20.38 43	69.62
08	6.91 16	83.09	6.86 16	83.14	58	22.35 54	67.65	20.82 44	69.18
09	7.07 16	82.93	7.01 16	82.99	59	22.90 56	67.10	21.26 45	68.74
10	7.23 17	82.77	7.18 16	82.82	60	23.46 57	66.54	21.71 45	68.29
11	7.40 17	82.60	7.34 17	82.66	61	24.04 59	65.96	22.17 46	67.83
12	7.58 18	82.42	7.51 17	82.49	62	24.64 61	65.36	22.63 47	67.37
13	7.75 18	82.25	7.68 17	82.32	63	25.25 62	64.75	23.10 48	69.90
14	7.93 18	82.07	7.86 18	82.14	64	25.88 64	64.12	23.58 48	66.42
15	8.12 19	81.88	8.04 18	81.96	65	26.53 66	63.47	24.07 49	65.93
16	8.31 19	81.69	8.22 19	81.78	66	27.20 68	62.80	24.56 50	65.44
17	8.51 20	81.49	8.41 19	81.59	67	27.89 70	62.11	25.07 51	64.93
18	8.71 20	81.29	8.61 20	81.39	68	28.60 72	61.40	25.58 51	64.42
19	8.91 21	81.09	8.80 20	81.20	69	29.33 74	60.67	26.09 52	63.91
20	9.12 21	80.88	9.01 20	80.99	70	30.08 76	59.92	26.62 53	63.38
21	9.33 22	80.67	9.21 21	80.79	71	30.85 79	59.15	27.15 54	62.85
22	9.55 22	80.45	9.42 21	80.58	72	31.66 81	58.34	27.69 54	62.31
23	9.78 23	80.22	9.64 22	80.36	73	32.48 84	57.52	28.24 55	61.76
24	10.01 23	79.99	9.86 22	80.14	74	33.34 87	56.66	28.79 56	61.21
25	10.24 24	79.76	10.08 23	79.92	75	34.22 90	55.78	29.35 56	60.65
26	10.48 24	79.52	10.31 23	79.69	76	35.13	54.87	29.92 57	60.08
27	10.73 25	79.27	10.55 24	79.45	77	36.07	53.93	30.49 58	59.51
28	10.98 26	79.02	10.79 24	79.21	78	37.05	52.95	31.07 58	58.93
29	11.24 26	78.76	11.03 25	78.97	79	38.07	51.93	31.66 59	58.34
30	11.51 27	78.49	11.28 25	78.72	80	39.12	50.88	32.25 60	57.75
31	11.78 28	78.22	11.54 26	78.46	81	40.21	49.79	32.85 60	57.15
32	12.06 28	77.94	11.80 26	78.20	82	41.35	48.65	33.45 61	56.55
33	12.34 29	77.66	12.07 27	77.93	83	42.54	47.46	34.06 61	55.94
34	12.64 30	77.36	12.34 28	77.66	84	43.78	46.22	34.68 62	55.32
35	12.94 30	77.06	12.62 28	77.38	85	45.07	44.93	35.30 62	54.70
36	13.24 31	76.76	12.90 29	77.10	86	46.42	43.58	35.92 63	54.08
37	13.56 32	76.44	13.19 29	76.81	87	47.84	42.16	36.55 63	53.45
38	13.88 33	76.12	13.49 30	76.51	88	49.34	40.66	37.18 64	52.82
39	14.21 33	75.79	13.79 31	76.21	89	50.92	39.08	37.82 64	52.18
40	14.55 34	75.45	14.10 31	75.90	90	52.59	37.41	38.46 64	51.54
41	14.89 35	75.11	14.42 32	75.58	91	54.37	35.63	39.11 65	50.89
42	15.25 36	74.75	14.74 32	75.26	92	56.28	33.72	39.75 65	50.25
43	15.61 37	74.39	15.06 33	74.94	93	58.34	31.66	40.40 65	49.60
44	15.99 38	74.01	15.40 34	74.60	94	60.57	29.43	41.05 65	48.95
45	16.37 39	73.63	15.74 34	74.26	95	63.03	26.97	41.71 66	48.29
46	16.76 40	73.24	16.09 35	73.91	96	65.78	24.22	42.37 66	47.63
47	17.16 41	72.84	16.44 36	73.56	97	68.95	21.05	43.02 66	46.98
48	17.58 42	72.42	16.80 37	73.20	98	72.74	17.26	43.68 66	46.32
49	18.00 43	72.00	17.17 37	72.83	99	77.75	12.25	44.34 66	45.66
50	18.43 44	71.57	17.55 38	72.45	00	90.00	00.00	45.00 66	45.00

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log u	$\sin^{-1} u$		$\cos^{-1} u$	log u	$\sin^{-1} u$		$\cos^{-1} u$	log u	$\sin^{-1} u$		$\cos^{-1} u$
9.	°		°	9.	°		°	9.	°		°
750	34.22	9	55.78	**800**	39.12	11	50.88	**850**	45.07	13	44.93
751	34.31	9	55.69	801	39.23	11	50.77	851	45.20	13	44.80
752	34.40	9	55.60	802	39.34	11	50.66	852	45.33	13	44.67
753	34.49	9	55.51	803	39.44	11	50.56	853	45.47	13	44.53
754	34.58	9	55.42	804	39.55	11	50.45	854	45.60	13	44.40
755	34.67	9	55.33	**805**	39.66	11	50.34	**855**	45.74	14	44.26
756	34.76	9	55.24	806	39.77	11	50.23	856	45.87	14	44.13
757	34.85	9	55.15	807	39.88	11	50.12	857	46.01	14	43.99
758	34.95	9	55.05	808	39.99	11	50.01	858	46.15	14	43.85
759	35.04	9	54.96	809	40.10	11	49.90	859	46.28	14	43.72
760	35.13	9	54.87	**810**	40.21	11	49.79	**860**	46.42	14	43.58
761	35.22	9	54.78	811	40.33	11	49.67	861	46.56	14	43.44
762	35.32	9	54.68	812	40.44	11	49.56	862	46.70	14	43.30
763	35.41	9	54.59	813	40.55	11	49.45	863	46.84	14	43.16
764	35.50	9	54.50	814	40.66	11	49.34	864	46.98	14	43.02
765	35.60	9	54.40	**815**	40.78	11	49.22	**865**	47.12	14	42.88
766	35.69	9	54.31	816	40.89	11	49.11	866	47.27	14	42.73
767	35.79	10	54.21	817	41.01	11	48.99	867	47.41	14	42.59
768	35.88	10	54.12	818	41.12	12	48.88	868	47.55	14	42.45
769	35.98	10	54.02	819	41.24	12	48.76	869	47.70	14	42.30
770	36.07	10	53.93	**820**	41.35	12	48.65	**870**	47.84	15	42.16
771	36.17	10	53.83	821	41.47	12	48.53	871	47.99	15	42.01
772	36.27	10	53.73	822	41.59	12	48.41	872	48.14	15	41.86
773	36.36	10	53.64	823	41.70	12	48.30	873	48.28	15	41.72
774	36.46	10	53.54	824	41.82	12	48.18	874	48.43	15	41.57
775	36.56	10	53.44	**825**	41.94	12	48.06	**875**	48.58	15	41.42
776	36.66	10	53.34	826	42.06	12	47.94	876	48.73	15	41.27
777	36.76	10	53.24	827	42.18	12	47.82	877	48.88	15	41.12
778	36.85	10	53.15	828	42.30	12	47.70	878	49.03	15	40.97
779	36.95	10	53.05	829	42.42	12	47.58	879	49.19	15	40.81
780	37.05	10	52.95	**830**	42.54	12	47.46	**880**	49.34	15	40.66
781	37.15	10	52.85	831	42.66	12	47.34	881	49.49	15	40.51
782	37.25	10	52.75	832	42.78	12	47.22	882	49.65	16	40.35
783	37.35	10	52.65	833	42.90	12	47.10	883	49.80	16	40.20
784	37.45	10	52.55	834	43.03	12	46.97	884	49.96	16	40.04
785	37.56	10	52.44	**835**	43.15	12	46.85	**885**	50.12	16	39.88
786	37.66	10	52.34	836	43.27	12	46.73	886	50.28	16	39.72
787	37.76	10	52.24	837	43.40	12	46.60	887	50.44	16	39.56
788	37.86	10	52.14	838	43.52	13	46.48	888	50.60	16	39.40
789	37.96	10	52.04	839	43.65	13	46.35	889	50.76	16	39.24
790	38.07	10	51.93	**840**	43.78	13	46.22	**890**	50.92	16	39.08
791	38.17	10	51.83	841	43.90	13	46.10	891	51.08	16	38.92
792	38.28	10	51.72	842	44.03	13	45.97	892	51.25	16	38.75
793	38.38	10	51.62	843	44.16	13	45.84	893	51.41	17	38.59
794	38.48	10	51.52	844	44.29	13	45.71	894	51.58	17	38.42
795	38.59	11	51.41	**845**	44.41	13	45.59	**895**	51.74	17	38.26
796	38.69	11	51.31	846	44.54	13	45.46	896	51.91	17	38.09
797	38.80	11	51.20	847	44.67	13	45.33	897	52.08	17	37.92
798	38.91	11	51.09	848	44.80	13	45.20	898	52.25	17	37.75
799	39.01	11	50.99	849	44.94	13	45.06	899	52.42	17	37.58
800	39.12	11	50.88	**850**	45.07	13	44.93	**900**	52.59	17	37.41

log u	$\sin^{-1}u$	$\cos^{-1}u$	log u	$\sin^{-1}u$	$\cos^{-1}u$	log u	$\sin^{-1}u$	log u	$\sin^{-1}u$
9.	°	°	9.	°	°	9.	°	9.	°
900	52.59 17	37.41	**950**	63.03 26	26.97	**9900**	77.75	**9950**	81.32
901	52.76 17	37.24	951	63.29 26	26.71	9901	77.81	9951	81.41
902	52.94 17	37.06	952	63.56 27	26.44	9902	77.87	9952	81.50
903	53.11 18	36.89	953	63.82 27	26.18	9903	77.94	9953	81.59
904	53.29 18	36.71	954	64.09 27	25.91	9904	78.00	9954	81.68
905	53.47 18	36.53	**955**	64.37 27	25.63	**9905**	78.06	**9955**	81.77
906	53.65 18	36.35	956	64.64 28	25.36	9906	78.12	9956	81.86
907	53.83 18	36.17	957	64.92 28	25.08	9907	78.18	9957	81.95
908	54.01 18	35.99	958	65.21 29	24.79	9908	78.25	9958	82.04
909	54.19 18	35.81	959	65.49 29	24.51	9909	78.31	9959	82.14
910	54.37 18	35.63	**960**	65.78 29	24.22	**9910**	78.38	**9960**	82.24
911	54.56 19	35.44	961	66.08 30	23.92	9911	78.44	9961	82.33
912	54.74 19	35.26	962	66.38 30	23.62	9912	78.50	9962	82.43
913	54.93 19	35.07	963	66.68 31	23.32	9913	78.57	9963	82.5
914	55.12 19	34.88	964	66.99 31	23.01	9914	78.64	9964	82.6
915	55.31 19	34.69	**965**	67.30 32	22.70	**9915**	78.70	**9965**	82.7
916	55.50 19	34.50	966	67.62 32	22.38	9916	78.77	9966	82.8
917	55.69 19	34.31	967	67.95 33	22.05	9917	78.83	9967	82.9
918	55.89 19	34.11	968	68.27 33	21.73	9918	78.90	9968	83.1
919	56.08 20	33.92	969	68.61 34	21.39	9919	78.97	9969	83.2
920	56.28 20	33.72	**970**	68.95 34	21.05	**9920**	79.04	**9970**	83.3
921	56.48 20	33.52	971	69.29 35	20.71	9921	79.10	9971	83.4
922	56.68 20	33.32	972	69.65 36	20.35	9922	79.17	9972	83.5
923	56.88 20	33.12	973	70.01 36	19.99	9923	79.24	9973	83.6
924	57.08 20	32.92	974	70.37 37	19.63	9924	79.31	9974	83.7
925	57.29 21	32.71	**975**	70.75 38	19.25	**9925**	79.38	**9975**	83.9
926	57.49 21	32.51	976	71.13 39	18.87	9926	79.45	9976	84.0
927	57.70 21	32.30	977	71.52 39	18.48	9927	79.52	9977	84.1
928	57.91 21	32.09	978	71.92 40	18.08	9928	79.60	9978	84.2
929	58.12 21	31.88	979	72.33 41	17.67	9929	79.67	9979	84.4
930	58.34 21	31.66	**980**	72.74 42	17.26	**9930**	79.74	**9980**	84.5
931	58.55 22	31.45	981	73.18 44	16.82	9931	79.81	9981	84.6
932	58.77 22	31.23	982	73.62 45	16.38	9932	79.89	9982	84.8
933	58.99 22	31.01	983	74.07 46	15.93	9933	79.96	9983	84.9
934	59.21 22	30.79	984	74.54 48	15.46	9934	80.04	9984	85.1
935	59.43 22	30.57	**985**	75.03 49	14.97	**9935**	80.11	**9985**	85.2
936	59.65 23	30.35	986	75.53 51	14.47	9936	80.19	9986	85.4
937	59.88 23	30.12	987	76.05 53	13.95	9937	80.26	9987	85.6
938	60.11 23	29.89	988	76.59 55	13.41	9938	80.34	9988	85.7
939	60.34 23	29.66	989	77.16 58	12.84	9939	80.42	9989	85.9
940	60.57 23	29.43	**990**	77.75 61	12.25	**9940**	80.50	**9990**	86.1
941	60.81 24	29.19	991	78.38	11.62	9941	80.58	9991	86.3
942	61.04 24	28.96	992	79.04	10.96	9942	80.66	9992	86.5
943	61.28 24	28.72	993	79.74	10.26	9943	80.74	9993	86.7
944	61.52 24	28.48	994	80.50	9.50	9944	80.82	9994	87.0
945	61.77 25	28.23	**995**	81.32	8.68	**9945**	80.90	**9995**	87.3
946	62.02 25	27.98	996	82.24	7.76	9946	80.98	9996	87.5
947	62.27 25	27.73	997	83.3	6.7	9947	81.07	9997	87.9
948	62.52 25	27.48	998	84.5	5.5	9948	81.15	9998	88.3
949	62.77 26	27.23	999	86.1	3.9	9949	81.24	9999	88.8
950	63.03 26	26.97	**000**	90.	0.	**9950**	81.32	**0000**	90.

x	gd x	x	gd x	x	l Sh x	l Ch x	l Th x
	°		°		0.	0.	9.
0.00	0.0000 5730	**0.50**	27.524 508	**1.00**	0701 57	1884 33	8817 24
0.01	0.5729 5729	0.51	28.031 506	1.01	0758 57	1917 33	8840 23
0.02	1.1458 5728	0.52	28.535 503	1.02	0815 56	1950 33	8864 23
0.03	1.7186 5727	0.53	29.037 501	1.03	0871 56	1984 34	8887 23
0.04	2.2912 5725	0.54	29.537 498	1.04	0927 56	2018 34	8909 22
0.05	2.8636 5722	**0.55**	30.034 496	**1.05**	0982 56	2051 34	8931 22
0.06	3.4357 5719	0.56	30.529 494	1.06	1038 55	2086 34	8952 21
0.07	4.0074 5716	0.57	31.021 491	1.07	1093 55	2120 34	8973 21
0.08	4.5788 5711	0.58	31.511 488	1.08	1148 55	2154 34	8994 20
0.09	5.1497 5706	0.59	31.998 486	1.09	1203 54	2189 35	9014 20
0.10	5.720 570	**0.60**	32.483 483	**1.10**	1257 54	2223 35	9034 19
0.11	6.290 570	0.61	32.965 481	1.11	1311 54	2258 35	9053 19
0.12	6.859 569	0.62	33.444 478	1.12	1365 54	2293 35	9072 19
0.13	7.428 568	0.63	33.921 475	1.13	1419 54	2328 35	9090 18
0.14	7.995 567	0.64	34.395 473	1.14	1472 53	2364 35	9108 18
0.15	8.562 567	**0.65**	34.867 470	**1.15**	1525 53	2399 36	9126 18
0.16	9.128 566	0.66	35.336 467	1.16	1578 53	2435 36	9144 17
0.17	9.694 565	0.67	35.802 465	1.17	1631 53	2470 36	9161 17
0.18	10.258 564	0.68	36.265 462	1.18	1684 52	2506 36	9177 17
0.19	10.821 563	0.69	36.726 459	1.19	1736 52	2542 36	9194 16
0.20	11.384 562	**0.70**	37.183 456	**1.20**	1788 52	2578 36	9210 16
0.21	11.945 561	0.71	37.638 454	1.21	1840 52	2615 37	9226 16
0.22	12.505 559	0.72	38.091 451	1.22	1892 52	2651 36	9241 15
0.23	13.063 558	0.73	38.540 448	1.23	1944 52	2688 37	9256 15
0.24	13.621 557	0.74	38.987 445	1.24	1995 51	2724 37	9271 15
0.25	14.177 556	**0.75**	39.431 443	**1.25**	2046 51	2761 37	9285 14
0.26	14.732 554	0.76	39.872 440	1.26	2098 51	2798 37	9300 14
0.27	15.285 553	0.77	40.310 437	1.27	2148 51	2835 37	9314 14
0.28	15.837 551	0.78	40.746 434	1.28	2199 51	2872 37	9327 14
0.29	16.388 550	0.79	41.179 431	1.29	2250 51	2909 37	9341 13
0.30	16.937 548	**0.80**	41.608 428	**1.30**	2300 50	2947 37	9354 13
0.31	17.484 546	0.81	42.035 426	1.31	2351 50	2984 38	9367 13
0.32	18.030 545	0.82	42.460 423	1.32	2401 50	3022 38	9379 12
0.33	18.573 543	0.83	42.881 420	1.33	2451 50	3059 38	9391 12
0.34	19.116 541	0.84	43.299 417	1.34	2501 50	3097 38	9404 12
0.35	19.656 540	**0.85**	43.715 414	**1.35**	2551 50	3135 38	9415 12
0.36	20.195 538	0.86	44.128 411	1.36	2600 50	3173 38	9427 11
0.37	20.732 536	0.87	44.537 408	1.37	2650 49	3211 38	9438 11
0.38	21.267 534	0.88	44.944 406	1.38	2699 49	3249 38	9450 11
0.39	21.800 532	0.89	45.348 403	1.39	2748 49	3288 38	9460 11
0.40	22.331 530	**0.90**	45.750 400	**1.40**	2797 49	3326 38	9471 11
0.41	22.859 528	0.91	46.148 397	1.41	2846 49	3365 39	9482 10
0.42	23.386 526	0.92	46.544 394	1.42	2895 49	3403 39	9492 10
0.43	23.911 524	0.93	46.936 391	1.43	2944 49	3442 39	9502 10
0.44	24.434 522	0.94	47.326 388	1.44	2993 49	3481 39	9512 10
0.45	24.955 519	**0.95**	47.713 386	**1.45**	3041 48	3520 39	9522 10
0.46	25.473 517	0.96	48.097 383	1.46	3090 48	3559 39	9531 9
0.47	25.989 515	0.97	48.478 380	1.47	3138 48	3598 39	9540 9
0.48	26.503 513	0.98	48.857 377	1.48	3186 48	3637 39	9549 9
0.49	27.015 510	0.99	49.232 374	1.49	3234 48	3676 39	9558 9
0.50	27.524 508	**1.00**	49.605 371	**1.50**	3282 48	3715 39	9567 9

x	l Sh x	l Ch x	l Th x	x	l Sh x	l Ch x	l Th x
	0.	0.	9.		0.	0.	9.
1.50	3282 48	3715 39	9567 9	**2.00**	6595 45	5754 42	9841 3
1.51	3330 48	3754 39	9576 8	2.01	6640 45	5796 42	9844 3
1.52	3378 48	3794 39	9584 8	2.02	6685 45	5838 42	9847 3
1.53	3426 48	3833 40	9592 8	2.03	6730 45	5880 42	9850 3
1.54	3474 48	3873 40	9601 8	2.04	6775 45	5922 42	9853 3
1.55	3521 48	3913 40	9608 8	**2.05**	6820 45	5964 42	9856 3
1.56	3569 47	3952 40	9616 8	2.06	6865 45	6006 42	9859 3
1.57	3616 47	3992 40	9624 8	2.07	6910 45	6048 42	9862 3
1.58	3663 47	4032 40	9631 7	2.08	6955 45	6090 42	9864 3
1.59	3711 47	4072 40	9639 7	2.09	6000 45	6132 42	9867 3
1.60	3758 47	4112 40	9646 7	**2.10**	6044 45	6175 42	9870 3
1.61	3805 47	4152 40	9653 7	2.11	6089 45	6217 42	9872 3
1.62	3852 47	4192 40	9660 7	2.12	6134 45	6259 42	9875 3
1.63	3899 47	4232 40	9666 7	2.13	6178 45	6301 42	9877 2
1.64	3946 47	4273 40	9673 7	2.14	6223 45	6343 42	9880 2
1.65	3992 47	4313 40	9679 6	**2.15**	6268 45	6386 42	9882 2
1.66	4039 47	4353 40	9686 6	2.16	6312 45	6428 42	9884 2
1.67	4086 47	4394 40	9692 6	2.17	6357 45	6470 42	9887 2
1.68	4132 47	4434 41	9698 6	2.18	6401 45	6512 42	9889 2
1.69	4179 46	4475 41	9704 6	2.19	6446 45	6555 42	9891 2
1.70	4225 46	4515 41	9710 6	**2.20**	6491 45	6597 42	9893 2
1.71	4272 46	4556 41	9716 6	2.21	6535 44	6640 42	9895 2
1.72	4318 46	4597 41	9721 6	2.22	6580 44	6682 42	9898 2
1.73	4364 46	4637 41	9727 5	2.23	6624 44	6724 42	9900 2
1.74	4411 46	4678 41	9732 5	2.24	6668 44	6767 42	9902 2
1.75	4457 46	4719 41	9738 5	**2.25**	6713 44	6809 42	9904 2
1.76	4503 46	4760 41	9743 5	2.26	6757 44	6852 42	9905 2
1.77	4549 46	4801 41	9748 5	2.27	6802 44	6894 43	9907 2
1.78	4595 46	4842 41	9753 5	2.28	6846 44	6937 43	9909 2
1.79	4641 46	4883 41	9758 5	2.29	6890 44	6979 43	9911 2
1.80	4687 46	4924 41	9763 5	**2.30**	6935 44	7022 43	9913 2
1.81	4733 46	4965 41	9767 5	2.31	6979 44	7064 43	9914 2
1.82	4778 46	5006 41	9772 5	2.32	7023 44	7107 43	9916 2
1.83	4824 46	5048 41	9776 4	2.33	7067 44	7150 43	9918 2
1.84	4870 46	5089 41	9781 4	2.34	7112 44	7192 43	9919 2
1.85	4915 46	5130 41	9785 4	**2.35**	7156 44	7235 43	9921 2
1.86	4961 46	5172 41	9789 4	2.36	7200 44	7278 43	9923 2
1.87	5007 46	5213 41	9794 4	2.37	7244 44	7320 43	9924 2
1.88	5052 46	5254 41	9798 4	2.38	7289 44	7363 43	9926 1
1.89	5098 45	5296 41	9802 4	2.39	7333 44	7406 43	9927 1
1.90	5143 45	5337 42	9806 4	**2.40**	7377 44	7448 43	9929 1
1.91	5188 45	5379 42	9810 4	2.41	7421 44	7491 43	9930 1
1.92	5234 45	5421 42	9813 4	2.42	7465 44	7534 43	9931 1
1.93	5279 45	5462 42	9817 4	2.43	7509 44	7577 43	9933 1
1.94	5324 45	5504 42	9821 4	2.44	7553 44	7619 43	9934 1
1.95	5370 45	5545 42	9824 4	**2.45**	7597 44	7662 43	9935 1
1.96	5415 45	5587 42	9828 3	2.46	7642 44	7705 43	9937 1
1.97	5460 45	5629 42	9831 3	2.47	7686 44	7748 43	9938 1
1.98	5505 45	5671 42	9834 3	2.48	7730 44	7791 43	9939 1
1.99	5550 45	5713 42	9838 3	2.49	7774 44	7833 43	9940 1
2.00	5595 45	5754 42	9841 3	**2.50**	7818 44	7876 43	9941 1

x	l Sh x	l Ch x	l Th x
	0.	0.	9.
2.50	7818 44	7876 43	9941
2.51	7862 44	7919 43	9943
2.52	7906 44	7962 43	9944
2.53	7950 44	8005 43	9945
2.54	7994 44	8048 43	9946
2.55	8038 44	8091 43	9947
2.56	8082 44	8134 43	9948
2.57	8126 44	8176 43	9949
2.58	8169 44	8219 43	9950
2.59	8213 44	8262 43	9951
2.60	8257 44	8305 43	9952
2.61	8301 44	8348 43	9953
2.62	8345 44	8391 43	9954
2.63	8389 44	8434 43	9955
2.64	8433 44	8477 43	9956
2.65	8477 44	8520 43	9957
2.66	8521 44	8563 43	9958
2.67	8564 44	8606 43	9958
2.68	8608 44	8649 43	9959
2.69	8652 44	8692 43	9960
2.70	8696 44	8735 43	9961
2.71	8740 44	8778 43	9962
2.72	8784 44	8821 43	9962
2.73	8827 44	8864 43	9963
2.74	8871 44	8907 43	9964
2.75	8915 44	8951 43	9965
2.76	8959 44	8994 43	9965
2.77	9003 44	9037 43	9966
2.78	9046 44	9080 43	9967
2.79	9090 44	9123 43	9967
2.80	9134 44	9166 43	9968
2.81	9178 44	9209 43	9969
2.82	9221 44	9252 43	9969
2.83	9265 44	9295 43	9970
2.84	9309 44	9338 43	9970
2.85	9353 44	9382 43	9971
2.86	9396 44	9425 43	9972
2.87	9440 44	9468 43	9972
2.88	9484 44	9511 43	9973
2.89	9527 44	9554 43	9973
2.90	9571 44	9597 43	9974
2.91	9615 44	9641 43	9974
2.92	9658 44	9684 43	9975
2.93	9702 44	9727 43	9975
2.94	9746 44	9770 43	9976
2.95	9789 44	9813 43	9976
2.96	9833 44	9856 43	9977
2.97	9877 44	9900 43	9977
2.98	9920 44	9943 43	9978
2.99	9964 44	9986 43	9978
3.00	1.0008 44	1.0029 43	9978

x	l Sh x	l Ch x	l Th x
3.0	1.0008 436	1.0029 432	9.9978
3.1	1.0444 436	1.0462 433	9.9982
3.2	1.0880 436	1.0894 433	9.9986
3.3	1.1316 435	1.1327 433	9.9988
3.4	1.1751 435	1.1761 433	9.9990
3.5	1.2186 435	1.2194 434	9.9992
3.6	1.2621 435	1.2628 434	9.9994
3.7	1.3056 435	1.3061 434	9.9995
3.8	1.3491 435	1.3495 434	9.9996
3.9	1.3925 435	1.3929 434	9.9996
4.0	1.4360 435	1.4363 434	9.9997
4.1	1.4795 435	1.4797 434	9.9998
4.2	1.5229 434	1.5231 434	9.9998
4.3	1.5664 434	1.5665 434	9.9998
4.4	1.6098 434	1.6099 434	9.9999
4.5	1.6532 434	1.6533 434	9.9999
4.6	1.6967 434	1.6968 434	9.9999
4.7	1.7401 434	1.7402 434	9.9999
4.8	1.7836 434	1.7836 434	9.9999
4.9	1.8270 434	1.8270 434	0.0000
5.0	1.8704 434	1.8705 434	0.0000
5.1	1.9139 434	1.9139 434	0.0000
5.2	1.9573 434	1.9573 434	0.0000
5.3	2.0007 434	2.0007 434	0.0000
5.4	2.0442 434	2.0442 434	0.0000
5.5	2.0876 434	2.0876 434	0.0000
5.6	2.1310 434	2.1310 434	0.0000
5.7	2.1744 434	2.1745 434	0.0000
5.8	2.2179 434	2.2179 434	0.0000
5.9	2.2613 434	2.2613 434	0.0000
6.0	2.3047 4343	2.3047 4343	0.0000
7.0	2.7390 4343	2.7390 4343	0.0000
8.0	3.1733 4343	3.1733 4343	0.0000
9.0	3.6076 4343	3.6076 4343	0.0000
10.0	4.0419 4343	4.0419 4343	0.0000

For higher values:

$\log \mathrm{Sh}\, x = \log \mathrm{Ch}\, x = x\mu - 0.301030$;

$\mathrm{Sh}^{-1} u = \mathrm{Ch}^{-1} u = (\log u + 0.3010)\, \mu^{-1}$.

n	$n\mu$	$n\mu^{-1}$	n
1	0.434294	2.302585	1
2	0.868589	4.605170	2
3	1.302883	6.907755	3
4	1.737178	9.210340	4
5	2.171472	11.512925	5
6	2.605767	13.815511	6
7	3.040061	16.118096	7
8	3.474356	18.420681	8
9	3.908650	20.723266	9
10	4.342945	23.025851	10

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ϕ	$\sin\phi$	$\cos\phi$	
0°	.000000	1.0000	**90°**
10′	.002909	1.0000	50′
20′	.005818	1.0000	40′
30′	.008727	1.0000	30′
40′	.011635	0.9999	20′
50′	.014544	0.9999	10′
1°	.017452	0.9998	**89°**
10′	.02036	0.9998	50′
20′	.02327	0.9997	40′
30′	.02618	0.9997	30′
40′	.02908	0.9996	20′
50′	.03199	0.9995	10′
2°	.03490	0.9994	**88°**
10′	.03781	0.9993	50′
20′	.04071	0.9992	40′
30′	.04362	0.9990	30′
40′	.04653	0.9989	20′
50′	.04943	0.9988	10′
3°	.05234	0.9986	**87°**
10′	.05524	0.9985	50′
20′	.05814	0.9983	40′
30′	.06105	0.9981	30′
40′	.06395	0.9980	20′
50′	.06685	0.9978	10′
4°	.06976	0.9976	**86°**
10′	.07266	0.9974	50′
20′	.07556	0.9971	40′
30′	.07846	0.9969	30′
40′	.08136	0.9967	20′
50′	.08426	0.9964	10′
5°	.08716	0.9962	**85°**
10′	.09005	0.9959	50′
20′	.09295	0.9957	40′
30′	.09585	0.9954	30′
40′	.09874	0.9951	20′
50′	.10164	0.9948	10′
6°	.10453	0.9945	**84°**
10′	.1074	0.9942	50′
20′	.1103	0.9939	40′
30′	.1132	0.9936	30′
40′	.1161	0.9932	20′
50′	.1190	0.9929	10′
7°	.1219	0.9925	**83°**
10′	.1248	0.9922	50′
20′	.1276	0.9918	40′
30′	.1305	0.9914	30′
			82°
	$\cos\theta$	$\sin\theta$	θ

ϕ	$\sin\phi$	$\cos\phi$	
7°			
30′	.1305	.9914	30′
40′	.1334	.9911	20′
50′	.1363	.9907	10′
8°	.1392	.9903	**82°**
10′	.1421	.9899	50′
20′	.1449	.9894	40′
30′	.1478	.9890	30′
40′	.1507	.9886	20′
50′	.1536	.9881	10′
9°	.1564	.9877	**81°**
10′	.1593	.9872	50′
20′	.1622	.9868	40′
30′	.1650	.9863	30′
40′	.1679	.9858	20′
50′	.1708	.9853	10′
10°	.1736	.9848	**80°**
10′	.1765	.9843	50′
20′	.1794	.9838	40′
30′	.1822	.9833	30′
40′	.1851	.9827	20′
50′	.1880	.9822	10′
11°	.1908	.9816	**79°**
10′	.1937	.9811	50′
20′	.1965	.9805	40′
30′	.1994	.9799	30′
40′	.2022	.9793	20′
50′	.2051	.9787	10′
12°	.2079	.9781	**78°**
10′	.2108	.9775	50′
20′	.2136	.9769	40′
30′	.2164	.9763	30′
40′	.2193	.9757	20′
50′	.2221	.9750	10′
13°	.2250	.9744	**77°**
10′	.2278	.9737	50′
20′	.2306	.9730	40′
30′	.2334	.9724	30′
40′	.2363	.9717	20′
50′	.2391	.9710	10′
14°	.2419	.9703	**76°**
10′	.2447	.9696	50′
20′	.2476	.9689	40′
30′	.2504	.9681	30′
40′	.2532	.9674	20′
50′	.2560	.9667	10′
15°	.2588	.9659	**75°**
	$\cos\theta$	$\sin\theta$	θ

ϕ	$\sin\phi$	$\cos\phi$	
15°	.2588	.9659	**75°**
10′	.2616	.9652	50′
20′	.2644	.9644	40′
30′	.2672	.9636	30′
40′	.2700	.9628	20′
50′	.2728	.9621	10′
16°	.2756	.9613	**74°**
10′	.2784	.9605	50′
20′	.2812	.9596	40′
30′	.2840	.9588	30′
40′	.2868	.9580	20′
50′	.2896	.9572	10′
17°	.2924	.9563	**73°**
10′	.2952	.9555	50′
20′	.2979	.9546	40′
30′	.3007	.9537	30′
40′	.3035	.9528	20′
50′	.3062	.9520	10′
18°	.3090	.9511	**72°**
10′	.3118	.9502	50′
20′	.3145	.9492	40′
30′	.3173	.9483	30′
40′	.3201	.9474	20′
50′	.3228	.9465	10′
19°	.3256	.9455	**71°**
10′	.3283	.9446	50′
20′	.3311	.9436	40′
30′	.3338	.9426	30′
40′	.3365	.9417	20′
50′	.3393	.9407	10′
20°	.3420	.9397	**70°**
10′	.3448	.9387	50′
20′	.3475	.9377	40′
30′	.3502	.9367	30′
40′	.3529	.9356	20′
50′	.3557	.9346	10′
21°	.3584	.9336	**69°**
10′	.3611	.9325	50′
20′	.3638	.9315	40′
30′	.3665	.9304	30′
40′	.3692	.9293	20′
50′	.3719	.9283	10′
22°	.3746	.9272	**68°**
10′	.3773	.9261	50′
20′	.3800	.9250	40′
30′	.3827	.9239	30′
			67°
	$\cos\theta$	$\sin\theta$	θ

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ϕ	$\sin\phi$	$\cos\phi$	
22°			
30′	.3827	.9239	30′
40′	.3854	.9228	20′
50′	.3881	.9216	10′
23°	.3907	.9205	**67°**
10′	.3934	.9194	50′
20′	.3961	.9182	40′
30′	.3987	.9171	30′
40′	.4014	.9159	20′
50′	.4041	.9147	10′
24°	.4067	.9135	**66°**
10′	.4094	.9124	50′
20′	.4120	.9112	40′
30′	.4147	.9100	30′
40′	.4173	.9088	20′
50′	.4200	.9075	10′
25°	.4226	.9063	**65°**
10′	.4253	.9051	50′
20′	.4279	.9038	40′
30′	.4305	.9026	30′
40′	.4331	.9013	20′
50′	.4358	.9001	10′
26°	.4384	.8988	**64°**
10′	.4410	.8975	50′
20′	.4436	.8962	40′
30′	.4462	.8949	30′
40′	.4488	.8936	20′
50′	.4514	.8923	10′
27°	.4540	.8910	**63°**
10′	.4566	.8897	50′
20′	.4592	.8884	40′
30′	.4617	.8870	30′
40′	.4643	.8857	20′
50′	.4669	.8843	10′
28°	.4695	.8829	**62°**
10′	.4720	.8816	50′
20′	.4746	.8802	40′
30′	.4772	.8788	30′
40′	.4797	.8774	20′
50′	.4823	.8760	10′
29°	.4848	.8746	**61°**
10′	.4874	.8732	50′
20′	.4899	.8718	40′
30′	.4924	.8704	30′
40′	.4950	.8689	20′
50′	.4975	.8675	10′
30°	.5000	.8660	**60°**
	$\cos\theta$	$\sin\theta$	θ

ϕ	$\sin\phi$	$\cos\phi$	
30°	.5000	.8660	**60°**
10′	.5025	.8646	50′
20′	.5050	.8631	40′
30′	.5075	.8616	30′
40′	.5100	.8601	20′
50′	.5125	.8587	10′
31°	.5150	.8572	**59°**
10′	.5175	.8557	50′
20′	.5200	.8542	40′
30′	.5225	.8526	30′
40′	.5250	.8511	20′
50′	.5275	.8496	10′
32°	.5299	.8480	**58°**
10′	.5324	.8465	50′
20′	.5348	.8450	40′
30′	.5373	.8434	30′
40′	.5398	.8418	20′
50′	.5422	.8403	10′
33°	.5446	.8387	**57°**
10′	.5471	.8371	50′
20′	.5495	.8355	40′
30′	.5519	.8339	30′
40′	.5544	.8323	20′
50′	.5568	.8307	10′
34°	.5592	.8290	**56°**
10′	.5616	.8274	50′
20′	.5640	.8258	40′
30′	.5664	.8241	30′
40′	.5688	.8225	20′
50′	.5712	.8208	10′
35°	.5736	.8192	**55°**
10′	.5760	.8175	50′
20′	.5783	.8158	40′
30′	.5807	.8141	30′
40′	.5831	.8124	20′
50′	.5854	.8107	10′
36°	.5878	.8090	**54°**
10′	.5901	.8073	50′
20′	.5925	.8056	40′
30′	.5948	.8039	30′
40′	.5972	.8021	20′
50′	.5995	.8004	10′
37°	.6018	.7986	**53°**
10′	.6041	.7969	50′
20′	.6065	.7951	40′
30′	.6088	.7934	30′
			52°
	$\cos\theta$	$\sin\theta$	θ

ϕ	$\sin\phi$	$\cos\phi$	
37°			
30′	.6088	.7934	30′
40′	.6111	.7916	20′
50′	.6134	.7898	10′
38°	.6157	.7880	**52°**
10′	.6180	.7862	50′
20′	.6202	.7844	40′
30′	.6225	.7826	30′
40′	.6248	.7808	20′
50′	.6271	.7790	10′
39°	.6293	.7771	**51°**
10′	.6316	.7753	50′
20′	.6338	.7735	40′
30′	.6361	.7716	30′
40′	.6383	.7698	20′
50′	.6406	.7679	10′
40°	.6428	.7660	**50°**
10′	.6450	.7642	50′
20′	.6472	.7623	40′
30′	.6494	.7604	30′
40′	.6517	.7585	20′
50′	.6539	.7566	10′
41°	.6561	.7547	**49°**
10′	.6583	.7528	50′
20′	.6604	.7509	40′
30′	.6626	.7490	30′
40′	.6648	.7470	20′
50′	.6670	.7451	10′
42°	.6691	.7431	**48°**
10′	.6713	.7412	50′
20′	.6734	.7392	40′
30′	.6756	.7373	30′
40′	.6777	.7353	20′
50′	.6799	.7333	10′
43°	.6820	.7314	**47°**
10′	.6841	.7294	50′
20′	.6862	.7274	40′
30′	.6884	.7254	30′
40′	.6905	.7234	20′
50′	.6926	.7214	10′
44°	.6947	.7193	**46°**
10′	.6967	.7173	50′
20′	.6988	.7153	40′
30′	.7009	.7133	30′
40′	.7030	.7112	20′
50′	.7050	.7092	10′
45°	.7071	.7071	**45°**
	$\cos\theta$	$\sin\theta$	θ

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ϕ	$\tan\phi$	$\operatorname{ctn}\phi$	
0°	.000000		**90°**
10′	.002909		50′
20′	.005818		40′
30′	.008727		30′
40′	.011636		20′
50′	.014545		10′
1°	.017455	57.	**89°**
10′	.02036	49.	50′
20′	.02328	43.	40′
30′	.02619	38.	30′
40′	.02910	34.	20′
50′	.03201	31.	10′
2°	.03492	28.6	**88°**
10′	.03783	26.4	50′
20′	.04075	24.5	40′
30′	.04366	22.9	30′
40′	.04658	21.5	20′
50′	.04949	20.2	10′
3°	.05241	19.1	**87°**
10′	.05533	18.1	50′
20′	.05824	17.2	40′
30′	.06116	16.3	30′
40′	.06408	15.6	20′
50′	.06700	14.9	10′
4°	.06993	14.3	**86°**
10′	.07285	13.73	50′
20′	.07578	13.20	40′
30′	.07870	12.71	30′
40′	.08163	12.25	20′
50′	.08456	11.83	10′
5°	.08749	11.43	**85°**
10′	.09042	11.06	50′
20′	.09335	10.71	40′
30′	.09629	10.39	30′
40′	.09923	10.08	20′
50′	.10216	9.79	10′
6°	.10510	9.51	**84°**
10′	.1080	9.26	50′
20′	.1110	9.01	40′
30′	.1139	8.78	30′
40′	.1169	8.56	20′
50′	.1198	8.34	10′
7°	.1228	8.14	**83°**
10′	.1257	7.95	50′
20′	.1287	7.77	40′
30′	.1317	7.60	30′
			82°
	$\operatorname{ctn}\theta$	$\tan\theta$	θ

ϕ	$\tan\phi$	$\operatorname{ctn}\phi$	
7°			
30′	.1317	7.60	30′
40′	.1346	7.43	20′
50′	.1376	7.27	10′
8°	.1405	7.12	**82°**
10′	.1435	6.97	50′
20′	.1465	6.83	40′
30′	.1495	6.69	30′
40′	.1524	6.56	20′
50′	.1554	6.43	10′
9°	.1584	6.31	**81°**
10′	.1614	6.197	50′
20′	.1644	6.084	40′
30′	.1673	5.976	30′
40′	.1703	5.871	20′
50′	.1733	5.769	10′
10°	.1763	5.671	**80°**
10′	.1793	5.576	50′
20′	.1823	5.485	40′
30′	.1853	5.396	30′
40′	.1883	5.309	20′
50′	.1914	5.226	10′
11°	.1944	5.145	**79°**
10′	.1974	5.066	50′
20′	.2004	4.989	40′
30′	.2035	4.915	30′
40′	.2065	4.843	20′
50′	.2095	4.773	10′
12°	.2126	4.705	**78°**
10′	.2156	4.638	50′
20′	.2186	4.574	40′
30′	.2217	4.511	30′
40′	.2247	4.449	20′
50′	.2278	4.390	10′
13°	.2309	4.331	**77°**
10′	.2339	4.275	50′
20′	.2370	4.219	40′
30′	.2401	4.165	30′
40′	.2432	4.113	20′
50′	.2462	4.061	10′
14°	.2493	4.011	**76°**
10′	.2524	3.962	50′
20′	.2555	3.914	40′
30′	.2586	3.867	30′
40′	.2617	3.821	20′
50′	.2648	3.776	10′
15°	.2679	3.732	**75°**
	$\operatorname{ctn}\theta$	$\tan\theta$	θ

ϕ	$\tan\phi$	$\operatorname{ctn}\phi$	
15°	.2679	3.732	**75°**
10′	.2711	3.689	50′
20′	.2742	3.647	40′
30′	.2773	3.606	30′
40′	.2805	3.566	20′
50′	.2836	3.526	10′
16°	.2867	3.487	**74°**
10′	.2899	3.450	50′
20′	.2931	3.412	40′
30′	.2962	3.376	30′
40′	.2994	3.340	20′
50′	.3026	3.305	10′
17°	.3057	3.271	**73°**
10′	.3089	3.237	50′
20′	.3121	3.204	40′
30′	.3153	3.172	30′
40′	.3185	3.140	20′
50′	.3217	3.108	10′
18°	.3249	3.078	**72°**
10′	.3281	3.047	50′
20′	.3314	3.018	40′
30′	.3346	2.989	30′
40′	.3378	2.960	20′
50′	.3411	2.932	10′
19°	.3443	2.904	**71°**
10′	.3476	2.877	50′
20′	.3508	2.850	40′
30′	.3541	2.824	30′
40′	.3574	2.798	20′
50′	.3607	2.773	10′
20°	.3640	2.747	**70°**
10′	.3673	2.723	50′
20′	.3706	2.699	40′
30′	.3739	2.675	30′
40′	.3772	2.651	20′
50′	.3805	2.628	10′
21°	.3839	2.605	**69°**
10′	.3872	2.583	50′
20′	.3906	2.560	40′
30′	.3939	2.539	30′
40′	.3973	2.517	20′
50′	.4006	2.496	10′
22°	.4040	2.475	**68°**
10′	.4074	2.455	50′
20′	.4108	2.434	40′
30′	.4142	2.414	30′
			67°
	$\operatorname{ctn}\theta$	$\tan\theta$	θ

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φ	tan φ	ctn φ	
22°			
30′	.4142	2.414	30′
40′	.4176	2.394	20′
50′	.4210	2.375	10′
23°	.4245	2.356	67°
10′	.4279	2.337	50′
20′	.4314	2.318	40′
30′	.4348	2.300	30′
40′	.4383	2.282	20′
50′	.4417	2.264	10′
24°	.4452	2.246	66°
10′	.4487	2.229	50′
20′	.4522	2.211	40′
30′	.4557	2.194	30′
40′	.4592	2.177	20′
50′	.4628	2.161	10′
25°	.4663	2.145	65°
10′	.4699	2.128	50′
20′	.4734	2.112	40′
30′	.4770	2.097	30′
40′	.4806	2.081	20′
50′	.4841	2.066	10′
26°	.4877	2.050	64°
10′	.4913	2.035	50′
20′	.4950	2.020	40′
30′	.4986	2.006	30′
40′	.5022	1.991	20′
50′	.5059	1.977	10′
27°	.5095	1.963	63°
10′	.5132	1.949	50′
20′	.5169	1.935	40′
30′	.5206	1.921	30′
40′	.5243	1.907	20′
50′	.5280	1.894	10′
28°	.5317	1.881	62°
10′	.5354	1.868	50′
20′	.5392	1.855	40′
30′	.5430	1.842	30′
40′	.5467	1.829	20′
50′	.5505	1.816	10′
29°	.5543	1.804	61°
10′	.5581	1.792	50′
20′	.5619	1.780	40′
30′	.5658	1.767	30′
40′	.5696	1.756	20′
50′	.5735	1.744	10′
30°	.5774	1.732	60°
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
30°	.5774	1.732	60°
10′	.5812	1.720	50′
20′	.5851	1.709	40′
30′	.5890	1.698	30′
40′	.5930	1.686	20′
50′	.5969	1.675	10′
31°	.6009	1.664	59°
10′	.6048	1.653	50′
20′	.6088	1.643	40′
30′	.6128	1.632	30′
40′	.6168	1.621	20′
50′	.6208	1.611	10′
32°	.6249	1.600	58°
10′	.6289	1.590	50′
20′	.6330	1.580	40′
30′	.6371	1.570	30′
40′	.6412	1.560	20′
50′	.6453	1.550	10′
33°	.6494	1.540	57°
10′	.6536	1.530	50′
20′	.6577	1.520	40′
30′	.6619	1.511	30′
40′	.6661	1.501	20′
50′	.6703	1.492	10′
34°	.6745	1.483	56°
10′	.6787	1.473	50′
20′	.6830	1.464	40′
30′	.6873	1.455	30′
40′	.6916	1.446	20′
50′	.6959	1.437	10′
35°	.7002	1.428	55°
10′	.7046	1.419	50′
20′	.7089	1.411	40′
30′	.7133	1.402	30′
40′	.7177	1.393	20′
50′	.7221	1.385	10′
36°	.7265	1.376	54°
10′	.7310	1.368	50′
20′	.7355	1.360	40′
30′	.7400	1.351	30′
40′	.7445	1.343	20′
50′	.7490	1.335	10′
37°	.7536	1.327	53°
10′	.7581	1.319	50′
20′	.7627	1.311	40′
30′	.7673	1.303	30′
	ctn θ	tan θ	θ

φ	tan φ	ctn φ	
37°			
30′	.7673	1.303	30′
40′	.7720	1.295	20′
50′	.7766	1.288	10′
38°	.7813	1.280	52°
10′	.7860	1.272	50′
20′	.7907	1.265	40′
30′	.7954	1.257	30′
40′	.8002	1.250	20′
50′	.8050	1.242	10′
39°	.8098	1.235	51°
10′	.8146	1.228	50′
20′	.8195	1.220	40′
30′	.8243	1.213	30′
40′	.8292	1.206	20′
50′	.8342	1.199	10′
40°	.8391	1.192	50°
10′	.8441	1.185	50′
20′	.8491	1.178	40′
30′	.8541	1.171	30′
40′	.8591	1.164	20′
50′	.8642	1.157	10′
41°	.8693	1.150	49°
10′	.8744	1.144	50′
20′	.8796	1.137	40′
30′	.8847	1.130	30′
40′	.8899	1.124	20′
50′	.8952	1.117	10′
42°	.9004	1.111	48°
10′	.9057	1.104	50′
20′	.9110	1.098	40′
30′	.9163	1.091	30′
40′	.9217	1.085	20′
50′	.9271	1.079	10′
43°	.9325	1.072	47°
10′	.9380	1.066	50′
20′	.9435	1.060	40′
30′	.9490	1.054	30′
40′	.9545	1.048	20′
50′	.9601	1.042	10′
44°	.9657	1.036	46°
10′	.9713	1.030	50′
20′	.9770	1.024	40′
30′	.9827	1.018	30′
40′	.9884	1.012	20′
50′	.9942	1.006	10′
45°	1.0000	1.000	45°
	ctn θ	tan θ	θ

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ϕ	sec ϕ	csc ϕ	
0°	1.0000		90°
10′	1.0000		50′
20′	1.0000		40′
30′	1.0000		30′
40′	1.0001		20′
50′	1.0001		10′
1°	1.0002	57.	89°
10′	1.0002	49.	50′
20′	1.0003	43.	40′
30′	1.0003	38.	30′
40′	1.0004	34.	20′
50′	1.0005	31.	10′
2°	1.0006	28.7	88°
10′	1.0007	26.5	50′
20′	1.0008	24.6	40′
30′	1.0010	22.9	30′
40′	1.0011	21.5	20′
50′	1.0012	20.2	10′
3°	1.0014	19.1	87°
10′	1.0015	18.1	50′
20′	1.0017	17.2	40′
30′	1.0019	16.4	30′
40′	1.0021	15.6	20′
50′	1.0022	15.0	10′
4°	1.0024	14.3	86°
10′	1.0026	13.76	50′
20′	1.0029	13.23	40′
30′	1.0031	12.75	30′
40′	1.0033	12.29	20′
50′	1.0036	11.87	10′
5°	1.0038	11.47	85°
10′	1.0041	11.10	50′
20′	1.0043	10.76	40′
30′	1.0046	10.43	30′
40′	1.0049	10.13	20′
50′	1.0052	9.84	10′
6°	1.0055	9.57	84°
10′	1.0058	9.31	50′
20′	1.0061	9.07	40′
30′	1.0065	8.83	30′
40′	1.0068	8.61	20′
50′	1.0072	8.40	10′
7°	1.0075	8.21	83°
10′	1.0079	8.02	50′
20′	1.0082	7.83	40′
30′	1.0086	7.66	30′
			82°
	csc θ	sec θ	θ

ϕ	sec ϕ	csc ϕ	
7°			
30′	1.009	7.66	30′
40′	1.009	7.60	20′
50′	1.009	7.34	10′
8°	1.010	7.19	82°
10′	1.010	7.04	50′
20′	1.011	6.90	40′
30′	1.011	6.77	30′
40′	1.012	6.64	20′
50′	1.012	6.51	10′
9°	1.012	6.39	81°
10′	1.013	6.277	50′
20′	1.013	6.166	40′
30′	1.014	6.059	30′
40′	1.014	5.955	20′
50′	1.015	5.855	10′
10°	1.015	5.759	80°
10′	1.016	5.665	50′
20′	1.016	5.575	40′
30′	1.017	5.487	30′
40′	1.018	5.403	20′
50′	1.018	5.320	10′
11°	1.019	5.241	79°
10′	1.019	5.164	50′
20′	1.020	5.089	40′
30′	1.020	5.016	30′
40′	1.021	4.945	20′
50′	1.022	4.876	10′
12°	1.022	4.810	78°
10′	1.023	4.745	50′
20′	1.024	4.682	40′
30′	1.024	4.620	30′
40′	1.025	4.560	20′
50′	1.026	4.502	10′
13°	1.026	4.445	77°
10′	1.027	4.390	50′
20′	1.028	4.336	40′
30′	1.028	4.284	30′
40′	1.029	4.232	20′
50′	1.030	4.182	10′
14°	1.031	4.134	76°
10′	1.031	4.086	50′
20′	1.032	4.039	40′
30′	1.033	3.994	30′
40′	1.034	3.950	20′
50′	1.034	3.906	10′
15°	1.035	3.864	75°
	csc θ	sec θ	θ

ϕ	sec ϕ	csc ϕ	
15°	1.035	3.864	75°
10′	1.036	3.822	50′
20′	1.037	3.782	40′
30′	1.038	3.742	30′
40′	1.039	3.703	20′
50′	1.039	3.665	10′
16°	1.040	3.628	74°
10′	1.041	3.592	50′
20′	1.042	3.556	40′
30′	1.043	3.521	30′
40′	1.044	3.487	20′
50′	1.045	3.453	10′
17°	1.046	3.420	73°
10′	1.047	3.388	50′
20′	1.048	3.356	40′
30′	1.049	3.326	30′
40′	1.049	3.295	20′
50′	1.050	3.265	10′
18°	1.051	3.236	72°
10′	1.052	3.207	50′
20′	1.053	3.179	40′
30′	1.054	3.152	30′
40′	1.056	3.124	20′
50′	1.057	3.098	10′
19°	1.058	3.072	71°
10′	1.059	3.046	50′
20′	1.060	3.021	40′
30′	1.061	2.996	30′
40′	1.062	2.971	20′
50′	1.063	2.947	10′
20°	1.064	2.924	70°
10′	1.065	2.901	50′
20′	1.066	2.878	40′
30′	1.068	2.855	30′
40′	1.069	2.833	20′
50′	1.070	2.812	10′
21°	1.071	2.790	69°
10′	1.072	2.769	50′
20′	1.074	2.749	40′
30′	1.075	2.729	30′
40′	1.076	2.709	20′
50′	1.077	2.689	10′
22°	1.079	2.669	68°
10′	1.080	2.650	50′
20′	1.081	2.632	40′
30′	1.082	2.613	30′
			67°
	csc θ	sec θ	θ

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φ	sec φ	csc φ	
22°			68°
30′	1.082	2.613	30′
40′	1.084	2.595	20′
50′	1.085	2.577	10′
23°	1.086	2.559	67°
10′	1.088	2.542	50′
20′	1.089	2.525	40′
30′	1.090	2.508	30′
40′	1.092	2.491	20′
50′	1.093	2.475	10′
24°	1.095	2.459	66°
10′	1.096	2.443	50′
20′	1.097	2.427	40′
30′	1.099	2.411	30′
40′	1.100	2.396	20′
50′	1.102	2.381	10′
25°	1.103	2.366	65°
10′	1.105	2.352	50′
20′	1.106	2.337	40′
30′	1.108	2.323	30′
40′	1.109	2.309	20′
50′	1.111	2.295	10′
26°	1.113	2.281	64°
10′	1.114	2.268	50′
20′	1.116	2.254	40′
30′	1.117	2.241	30′
40′	1.119	2.228	20′
50′	1.121	2.215	10′
27°	1.122	2.203	63°
10′	1.124	2.190	50′
20′	1.126	2.178	40′
30′	1.127	2.166	30′
40′	1.129	2.154	20′
50′	1.131	2.142	10′
28°	1.133	2.130	62°
10′	1.134	2.118	50′
20′	1.136	2.107	40′
30′	1.138	2.096	30′
40′	1.140	2.085	20′
50′	1.142	2.074	10′
29°	1.143	2.063	61°
10′	1.145	2.052	50′
20′	1.147	2.041	40′
30′	1.149	2.031	30′
40′	1.151	2.020	20′
50′	1.153	2.010	10′
30°	1.155	2.000	60°
	csc θ	sec θ	θ

φ	sec φ	csc φ	
30°	1.155	2.000	60°
10′	1.157	1.990	50′
20′	1.159	1.980	40′
30′	1.161	1.970	30′
40′	1.163	1.961	20′
50′	1.165	1.951	10′
31°	1.167	1.942	59°
10′	1.169	1.932	50′
20′	1.171	1.923	40′
30′	1.173	1.914	30′
40′	1.175	1.905	20′
50′	1.177	1.896	10′
32°	1.179	1.887	58°
10′	1.181	1.878	50′
20′	1.184	1.870	40′
30′	1.186	1.861	30′
40′	1.188	1.853	20′
50′	1.190	1.844	10′
33°	1.192	1.836	57°
10′	1.195	1.828	50′
20′	1.197	1.820	40′
30′	1.199	1.812	30′
40′	1.202	1.804	20′
50′	1.204	1.796	10′
34°	1.206	1.788	56°
10′	1.209	1.781	50′
20′	1.211	1.773	40′
30′	1.213	1.766	30′
40′	1.216	1.758	20′
50′	1.218	1.751	10′
35°	1.221	1.743	55°
10′	1.223	1.736	50′
20′	1.226	1.729	40′
30′	1.228	1.722	30′
40′	1.231	1.715	20′
50′	1.233	1.708	10′
36°	1.236	1.701	54°
10′	1.239	1.695	50′
20′	1.241	1.688	40′
30′	1.244	1.681	30′
40′	1.247	1.675	20′
50′	1.249	1.668	10′
37°	1.252	1.662	53°
10′	1.255	1.655	50′
20′	1.258	1.649	40′
30′	1.260	1.643	30′
			52°
	csc θ	sec θ	θ

φ	sec φ	csc φ	
37°			53°
30′	1.260	1.643	30′
40′	1.263	1.636	20′
50′	1.266	1.630	10′
38°	1.269	1.624	52°
10′	1.272	1.618	50′
20′	1.275	1.612	40′
30′	1.278	1.606	30′
40′	1.281	1.601	20′
50′	1.284	1.595	10′
39°	1.287	1.589	51°
10′	1.290	1.583	50′
20′	1.293	1.578	40′
30′	1.296	1.572	30′
40′	1.299	1.567	20′
50′	1.302	1.561	10′
40°	1.305	1.556	50°
10′	1.309	1.550	50′
20′	1.312	1.545	40′
30′	1.315	1.540	30′
40′	1.318	1.535	20′
50′	1.322	1.529	10′
41°	1.325	1.524	49°
10′	1.328	1.519	50′
20′	1.332	1.514	40′
30′	1.335	1.509	30′
40′	1.339	1.504	20′
50′	1.342	1.499	10′
42°	1.346	1.494	48°
10′	1.349	1.490	50′
20′	1.353	1.485	40′
30′	1.356	1.480	30′
40′	1.360	1.476	20′
50′	1.364	1.471	10′
43°	1.367	1.466	47°
10′	1.371	1.462	50′
20′	1.375	1.457	40′
30′	1.379	1.453	30′
40′	1.382	1.448	20′
50′	1.386	1.444	10′
44°	1.390	1.440	46°
10′	1.394	1.435	50′
20′	1.398	1.431	40′
30′	1.402	1.427	30′
40′	1.406	1.423	20′
50′	1.410	1.418	10′
45°	1.414	1.414	45°
	csc θ	sec θ	θ

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EXPLANATION OF THE TABLES.

§ 1. TABLES IN GENERAL.

a. One quantity is said to be a **function** of another, when the former quantity is regarded as determined by the latter, according to some rule or formula. E. g. x^2, $\sqrt{x}$, $\log x$, $\sin x$, $\log \sin x$, are all called functions of x. **A mathematical table** is an orderly arrangement of the values of some function for certain selected values of the quantity by which it is regarded as determined. The successive values of the latter quantity are assumed arbitrarily, and generally at equal intervals; and this quantity is called the **argument** of the table. Some functions require several independent quantities for their determination; and the corresponding tables are tables of several arguments. Thus, a multiplication-table is a table of *two* arguments; namely, the two factors.

b. A table may be used in two ways: **directly** and **inversely**. The direct use of the table consists in finding the value of the function for an assumed value of the argument; the inverse use, in finding the value of the argument for an assumed value of the function.

c. Before beginning to use any table, the student should give attentive consideration to its arrangement, and to the best mode of employing it with accuracy and ease. Every feature of it should be carefully examined, and the explanations which are attached to it should be fully mastered. The time thus spent will be time gained, contributing not only to power in computation, but also, very materially, to the thorough practical knowledge of the nature of the tabulated functions.

§ 2. INTERPOLATION.

a. **Interpolation** consists in finding the value of one of the two quantities, argument and function, for an assumed value of the other quantity, lying between two successive tabulated values. Most mathematical tables are so constructed as to admit of interpolation by the principle that *corresponding non-tabulated values of the function and argument lie between corresponding tabulated values and divide the differences between them in the same ratio.* This is the principle of **proportional parts**. Let x_1 and x_2 be two successive tabulated values of the argument of a table, and u_1 and u_2 the correspond-

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ing values of the function. Then, $x_2 - x_1$ and $u_2 - u_1$ are called corresponding tabular differences. We shall denote these differences by Δx and Δu. If, now, x and u are corresponding values of the function and argument, of which one is known to lie between the two above-cited tabulated values of the same quantity, the principle of proportional parts is that if

$$\lambda = \frac{x - x_1}{\Delta x}, \qquad \lambda' = \frac{x_2 - x}{\Delta x} = 1 - \lambda,$$

$$\mu = \frac{u - u_1}{\Delta u}, \qquad \mu' = \frac{u_2 - u}{\Delta u} = 1 - \mu,$$

then (to the limit of accuracy belonging to the table)

$$\lambda = \mu, \qquad \lambda' = \mu',$$

or,

$$u = u_1 + \lambda \Delta u = u_2 - \lambda' \Delta u,$$
$$x = x_1 + \mu \Delta x = x_2 - \mu' \Delta x.$$

Thus, the required value of the function or argument may be obtained by applying a **correction** to either of the two tabulated values between which the required value lies. In computing this correction, the *signs* of the differences employed must be carefully observed. If x_1 and x_2 are so chosen as to make Δx positive, Δu may be either positive or negative. In the former case, the function is said to be *increasing*; in the latter, *decreasing*.

b. Either of the two formulas given above for finding u may be employed, in interpolation, in the direct use of the table; either of the formulas for x may be employed in the inverse use of the table. In most tables, $\Delta x =$ *one unit* in the last numeral place of the tabulated values of x. Hence λ is composed of *the figures which follow that numeral place* in the given non-tabulated value of the argument, preceded by a decimal-point; while λ' is the *complement* of λ (that is, can be found by subtracting from 9 each figure of λ except the last, and subtracting that from 10). The correction for u is, therefore, found simply by *multiplying* the figures in question into Δu, and pointing off according to the case; x will be corrected by *annexing* to x_1 the figures of $\frac{u - u_1}{\Delta u}$, or the figures complementary to $\frac{u_2 - u}{\Delta u}$.

c. In some of the tables of this collection will be found, set against each value of the function, *a number in small type*, which shows what Δu would be if the function varied through a whole interval corresponding to Δx at the same rate at which it is changing when it passes through the value against which this number is set. This number may be called the *rate of difference*, or simply the *difference*, of u, and may be substituted for Δu in the formulas of interpolation. But, in that case, we ought to work from the NEAREST tabulated value of x or u; that is, from x_1 or u_1 when λ or $\mu < 0.5$, and from x_2 or u_2 when λ' or $\mu' < 0.5$. (See examples in the explanation of the table of *Logarithms of Circular Functions*.)

d. An interpolated value of the function should not be carried out beyond the last numeral place of the tabulated value from which it is computed; so that, in finding $\lambda \Delta u$ or $\lambda' \Delta u$, we should reject the decimal part of the product, Δu being regarded as an integer. Owing to the combination of the figures rejected in the correction and those omitted in the tabulated value of the function, an interpolated value is liable to an error of ± 1 in the last figure.

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The number of figures annexed to the tabulated value of the argument, in inverse interpolation, should be less by one than the number of figures contained in Δu. It is sometimes, indeed, made *equal* to the latter number (and will always be, if Δu consists of only one figure); but, in that case, the *last figure* must be regarded as *uncertain*. When the given value of the function is the result of computation, of course this uncertainty may extend back to earlier figures.

e. In taking the correction of either the function or the argument only to a certain number of figures, we must observe the following rule, which is a universal rule of computation:—

Whenever figures are neglected at the end of a number, if the figures neglected amount to more than half a unit in the place of the last figure retained, the last figure retained must be increased by 1. E. g. 27.528 = 27.53 to the nearest hundredth = 27.5 to the nearest tenth = 28 to the nearest unit = 30 to the nearest ten.

f. The various rules of interpolation will be found to be fully exemplified below, in the explanations of the tables of *Logarithms* and *Logarithms of Circular Functions*.

g. In interpolating in some tables (e. g. in VLACQ's great ten-place table), we must have regard to **second differences**, or differences between differences. In this case, we add to the above formulas for u the term

$$-\tfrac{1}{2}\lambda\lambda'\Delta^2 u,$$

where $\Delta^2 u$ denotes the second difference of u, taken positively when Δu is increasing. The greatest value of this term is *one eighth* of $\Delta^2 u$, so that it is insignificant when $\Delta^2 u < 4$. *In the present tables this term may always be neglected;* although it is useful as measuring the extent of error, and may occasionally guide the judgment of the computer when the fractional part of the correction is equal, or nearly equal, to 0.5. But where such nicety of work seems to be called for, it is best to use a table of a larger number of places.

§ 3. PROPORTIONAL PARTS.

a. The table of **Proportional Parts** (folded page) may be used in connexion with any other table, as an aid in *interpolation*. It contains the product of every integer from 1 to 100 by every *tenth* from 0.1 to 0.9. If the multiplier consists of one figure in any other numeral place, it is only necessary to change the position of the decimal-point in the product. To multiply a number of two figures by any decimal whatever, we must find the products which correspond to the successive figures of the multiplier, and add them together. The decimal part of the result is generally to be discarded, and in that case the general rule given above (in § 2, *e*) must be observed. Thus, let it be required to find 0.619×37. Looking in the column belonging to 37, we find

$$\begin{aligned} 0.6 \times 37 &= 22.2 \\ 0.01 \times 37 &= 0.37 \\ 0.009 \times 37 &= 0.333 \\ \therefore\ 0.619 \times 37 &= 23. \end{aligned}$$

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In like manner, we find

$0.27 \times 15 = 4,$	$0.59 \times 73 = 43,$	$0.78 \times 69 = 54,$
$0.96 \times 84 = 81,$	$0.36 \times 57 = 21,$	$0.289 \times 51 = 15,$
$0.483 \times 93 = 45,$	$0.374 \times 82 = 31,$	$0.053 \times 68 = 4.$

b. This table can also be used *inversely*. Thus, let it be required to find, to two decimal-places, what part 36 is of 79. Looking in the column of 79, we find

$$0.4 \times 79 = \begin{matrix} 36 \\ 31.6 \\ \hline 4.4 \end{matrix}$$

$$0.06 \times 79 = 4.74 \text{ (the } \textit{nearest} \text{ product)}$$

$$\therefore \frac{36}{79} = 0.46.$$

In like manner, we find

$$\frac{29}{68} = 0.43, \quad \frac{72}{89} = 0.81, \quad \frac{31}{98} = 0.32, \quad \frac{26}{71} = 0.37, \quad \frac{45}{57} = 0.79, \quad \frac{11}{37} = 0.30.$$

A little practice will enable the student to use this table easily and rapidly.

§ 4. LOGARITHMS.

a. **Denary, or Briggsian**, logarithms, being those employed in actual computation, are always referred to, in this collection of tables, when the term *logarithm* is used without qualification. The **characteristic**, or integral part, of the denary logarithm of a number depends only on the position of the first significant figure of the number relatively to the units' place, and may be found by a well-known rule; the **mantissa**, or fractional part, depends only on the series of significant figures which compose the number, and is the only part of the logarithm for which it is necessary to employ a table. A table of logarithms is complete, to an assigned number of places, if it gives (explicitly or by interpolation) to that number of places the mantissa of the logarithm of every possible series of significant figures. Denary logarithms are, in general, incommensurable numbers, and cannot, therefore, be exactly expressed in figures. They are variously given, in different tables, to ten, seven, six, five, four, and three places of decimals. Four-place logarithms are sufficient for the ordinary purposes of engineering, navigation, the work of the physical and chemical laboratory, and many of the subordinate computations of astronomy; and, in most of these cases, are all that the accuracy of the data will justify us in using. Seven places are, however, needful for the more accurate kinds of astronomical and geodetic work.

b. If one number is the logarithm of another, the second number is called the **antilogarithm** of the first. This relation is denoted by the symbol $\log^{-1}$. Thus, if $u = \log x$, then $x = \log^{-1} u$. In an ordinary table of logarithms, the *argument* is the antilogarithm, which is tabulated to a greater or less number of figures, according to the number of places to

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which the logarithm is given, and the *function* is the mantissa of the logarithm, which we often speak of simply as the *logarithm*.

To find the logarithm of any number.

c. If the number consists of *three significant figures*, seek the first two significant figures in the first column of the table of **Logarithms** (pp. 2, 3), and the third at the top of the table. In the line and column thus determined will be found the mantissa of the required logarithm, printed without the decimal-point. Find the characteristic by the rule, and prefix it, with the decimal-point, to the mantissa. E. g., $\log 2870 = 3.4579$. If the given number has *less than three* significant figures, fill it out to three figures by anexing a zero or zeros. E. g., $\log 0.35 = \log 0.350 = 9.5441 - 10$, $\log 6 = \log 6.00 = 0.7782$. If the number has *more than three* significant figures, its logarithm must be found by one of the formulas of interpolation given above. The rule is: — *Find the logarithm of the first three significant figures of the given number and also that of the next following number of three figures* (1000 following 999); *then apply to* EITHER *of these two logarithms a correction, obtained by multiplying the difference between them by the difference between the given number and the three-figure number which corresponds to the logarithm chosen to be corrected, and rejecting (with due attention to the rule of* § 2, *e) as many figures at the end of the product as are contained in the latter difference.* The table of **Proportional Parts** may be employed in performing the multiplications. Thus, to find $\log 5668.4$. Using the notation of the formulas of interpolation, and remembering that the place of the decimal-point in the given number may be disregarded in finding the mantissa of the required logarithm, we have

$$\begin{aligned} x_1 &= 566, & u_1 &= \text{mant} \log x_1 = 7528, \\ x_2 &= 567, & u_2 &= \text{mant} \log x_2 = 7536, \\ \Delta x &= 1, & \Delta u &= 8; \end{aligned}$$

so that $\log x$ may be found by either of the following methods: —

$\lambda = 0.84$, $\quad \lambda \Delta u = 6.72 = 7$ to units, $\quad u = 7528 + 7 = 7535$;

or,

$\lambda' = 1 - \lambda = 0.16$, $\quad \lambda' \Delta u = 1.28 = 1$ to units, $\quad u = 7536 - 1 = 7535$.

$$\therefore \log 5668.4 = 3.7535.$$

Let the beginner find the following logarithms by this method: —

$\log 59.43$	$= 1.7740$,	$\log 0.0081472$	$= 7.9110 - 10$,
$\log 284.8$	$= 2.4545$,	$\log 572820$	$= 5.7581$,
$\log 0.073748$	$= 8.8678 - 10$,	$\log 0.50167$	$= 9.7004 - 10$,
$\log 3.1607$	$= 0.4998$,	$\log 99968.$	$= 4.9999$.

The interpolated logarithm should never be carried to more than four decimal-places.

d. The work of interpolation may be shortened by using the column of proportional parts, marked P. P., on the right of the table. In using this column, one must work from the three-figure number NEAREST to the given

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number (in the above example, from 567, not from 566). If the given number has only four figures, so that λ or λ' has only one figure, then the correction will be found in the column P. P., under λ or λ' (according as we are working from the number below or the number above the given number), and in the same line with the logarithm to be corrected. If the given number has more than four figures, the correction must be estimated by the observation of the corrections which correspond to the figures below and above the first figure of λ or λ'. E. g.

mant log 2848 = mant log 285 — cor. for .2 = 4548 — 3 = 4545;
mant log 56684 = mant log 567 — cor. for .16 = 7536 — 2 = 7534.

In the last case the correction is either 1 or 2, and, since .16 is nearer .20 than .10, we choose the correction belonging to .20. Larger tables show that the mantissa of the required logarithm, to five places, is 75346; so that the value found by the column P. P. is here nearly as accurate as that obtained by computation. There is a slightly greater liability to error when we use the column P. P. than when we interpolate by computation; but the disadvantage is generally insignificant. The last figure of an interpolated logarithm obtained from any table may always be one unit in error. E. g.: the true mant log of 57282 to five places is 75802; and this is a case in which the column P. P. gives a better result than computation.

The student is advised now to find all the logarithms in the above list by using the column P. P.

e. *If the first figure of the given number is* 1, it will be found tabulated to *four* figures in pp. 4, 5. The correction for a fifth and following figures may be found by the method of interpolation explained in *c*. As the differences are always small on these pages, and the corrections easily computed, the column P. P. is not here given; but, to facilitate taking the last difference, we have printed at the end of each line, under the heading 10, the first logarithm of the following line. Let the student find the following logarithms: —

log 11.737 = 1.0696, log 0.00100066 = 7.0003 — 10,
log 0.15703 = 9.1960 — 10, log 18597. = 4.2694.

To find the antilogarithm of any logarithm.

f. It is enough to explain the way of finding the series of significant figures which compose the antilogarithm, by means of the mantissa of the given logarithm; the pointing off of the antilogarithm being determined, according to rule, by the given characteristic. If the mantissa of the given logarithm is contained in the table, the required antilogarithm is at once found by inspection. Otherwise, we must resort to the formulas of interpolation, which give the following rule: — *Find two successive tabulated logarithms* (u_1 and u_2) *between which the given logarithm* (u) *lies; then divide the difference between either of these tabulated logarithms and the given logarithm* ($u - u_1$ or $u_2 - u$) *by the difference between the tabulated logarithms* (Δu), *carry out the quotient to the* NEAREST *tenth* (that is, to one figure, which may be 0), *and add it to or subtract it from the antilogarithm* (x_1 or x_2) *of the tabulated logarithm* (u_1 or u_2) *with which the given logarithm has been compared.* The antilogarithm is always a figure *annexed* to the three or four tabulated figures of x_1.

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The division should not generally be carried beyond one figure. Even the first figure is, in most cases, somewhat uncertain. If the mantissa of the given logarithm is less than 3010, pp. 4, 5 should be used. On pp. 2, 3, the column P. P. may be employed.

Let it be required to find $\log^{-1} 1.5284$. We find

$$u_1 = 5276, \quad x_1 = 337, \quad u - u_1 = 8, \quad u_2 - u = 5,$$

$$u_2 = 5289, \quad x_2 = 338, \quad \mu = \mu \Delta x = \frac{8}{13} = 0.6\ldots,$$

$$\Delta u = 13, \quad \Delta x = 1, \quad \mu' = \mu' \Delta x = \frac{5}{13} = 0.4\ldots,$$

$$x = 337 + 0.6 = 338 - 0.4 = 337.6;$$

$$\therefore \log^{-1} 1.5284 = 33.76.$$

More briefly, looking along the line of $u_2 = 5289$ for $5 = u_2 - u$ in column P. P., we find that 5 corresponds to the correction 4, which gives at once the required number. In like manner, the student may find

$\log^{-1} 1.9155 = 82.32$, $\log^{-1} (5.8760 - 10) = 0.00007517$,
$\log^{-1} 3.8291 = 6747$, $\log^{-1} (9.5727 - 10) = 0.3738$,
$\log^{-1} 0.1548 = 1.4283$, $\log^{-1} (8.2731 - 10) = 0.018755$.

g. The convenient usage of *making negative characteristics positive*, by the addition of 10, is followed, throughout the present collection of tables, whenever logarithms are printed with their characteristics. This must be always understood, though no explicit reference be made to it in the explanation of the table.

§ 5. LOGARITHMS OF SUMS AND DIFFERENCES.

a. This is one form of a table devised by GAUSS to facilitate finding the logarithm of the *sum* or *difference* of two numbers which are themselves given only by their logarithms. The *argument* of the table is any logarithm, and may be called $\log x$; the *function* tabulated is then $\log (x + 1)$. It follows that, if the function is denoted by $\log x$, the argument is $\log (x - 1)$. The function may be called the **Gaussian** of the argument, and the argument the **anti-Gaussian** of the function; and the symbols $\mathfrak{G}$ and $\mathfrak{G}^{-1}$ may be used to denote these relations. Thus we have

$$\log (x + 1) = \mathfrak{G} \log x, \qquad \log (x - 1) = \mathfrak{G}^{-1} \log x.$$

b. **To find the Gaussian of a given logarithm.** Seek the characteristic of the given logarithm (increased by 10 if negative) at the top of the table, and the first two figures of the mantissa in the left-hand column. If the third and fourth figures of the mantissa are zero, the Gaussian will be found in the column and line thus determined; otherwise, it can be obtained by the method of interpolation which has been fully explained in § 2. In three columns of the table, the *rate of difference* of the Gaussian is printed in small type after the value of the function, and may be used instead of the tabular difference of the Gaussian *through half the tabular interval before and after* the value to which it is attached, as explained in

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§ 2, *c*, and completely illustrated below, in the explanation of the table of *Logarithms of Circular Functions*. The table of Proportional Parts may be employed in computing the corrections. Examples: —

$$\mathfrak{G}\ 1.0960 = 1.1295, \qquad \mathfrak{G}\ (7.5265 - 10) = 0.0015,$$
$$\mathfrak{G}\ 3.8129 = 3.8130, \qquad \mathfrak{G}\ (9.6431 - 10) = 0.1582.$$

If $\log x < 6.0000 - 10$, $\log (x + 1) = 0.0000$ to four places;
if $\log x > 4.0000$, $\log (x + 1) = \log x$ to four places.

c. **To find the anti-Gaussian of a given logarithm.** Seek, in the body of the table, two successive logarithms between which the given logarithm lies, and then find the corresponding value of the argument by interpolation. Examples: —

$$\mathfrak{G}^{-1}\ 1.0960 = 1.0597, \qquad \mathfrak{G}^{-1}\ 0.1051 = 9.4373 - 10,$$
$$\mathfrak{G}^{-1}\ 3.8129 = 3.8128, \qquad \mathfrak{G}^{-1}\ 1.0216 = 0.9782.$$

d. **To find the logarithm of the sum or difference of the anti-logarithms of two given logarithms.** If m and n are two numbers,

$$m + n = n\left(\frac{m}{n} + 1\right), \qquad m - n = n\left(\frac{m}{n} - 1\right),$$

$$\log (m + n) = \log n + \log\left(\frac{m}{n} + 1\right) = \log n + \mathfrak{G} \log \frac{m}{n},$$

$$\log (m - n) = \log n + \log\left(\frac{m}{n} - 1\right) = \log n + \mathfrak{G}^{-1} \log \frac{m}{n}.$$

Example: —

Given $a = 4.142$, $b = 2.399$; to find $\sqrt{(a^2 + b^2)}$ and $\sqrt{(a^2 - b^2)}$.

$\log a$	$= 0.6172$,	$\log b$	$= 0.3800$,
$\log a^2$	$= 1.2344$,	$\log b^2$	$= 0.7600$,
$\log \frac{a^2}{b^2}$	$= 0.4744$;		
$\mathfrak{G} \log \frac{a^2}{b^2}$	$= 0.6000$,	$\mathfrak{G}^{-1} \log \frac{a^2}{b^2}$	$= 0.2970$,
$\log b^2$	$= 0.7600$,	$\log b^2$	$= 0.7600$,
$\log (a^2 + b^2)$	$= 1.3600$,	$\log (a^2 - b^2)$	$= 1.0570$,
$\log \sqrt{(a^2 + b^2)}$	$= 0.6800$,	$\log \sqrt{(a^2 - b^2)}$	$= 0.5285$,
$\sqrt{(a^2 + b^2)}$	$= 4.787$;	$\sqrt{(a^2 - b^2)}$	$= 3.377$.

§ 6. CIRCULAR, OR TRIGONOMETRIC, FUNCTIONS: NATURAL VALUES.

a. Three tables of the natural values of the trigonometric functions are given on pp. 22-27. Each table is broken up into six divisions, and occupies two pages. The argument is the angle, which is tabulated at intervals of 10′ from 0° to 90°. Angles in the *first half of the quadrant* will be found in the *left-hand column* of the several divisions of the table, and for those angles the names of the functions are to be taken from the *top* of the page; angles in the *second half of the quadrant* are to be found in the *right-hand*

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column of the table, and for those angles the names of the functions are to be taken from the *bottom* of the page. The *angles* standing at the right and left in the same line are *complements* of each other; and the names of the *functions* at the top and bottom of the same column are *complementary*. The value of any of the functions for a non-tabulated angle, or the value of the angle for a non-tabulated value of one of the functions, can be found by the method of interpolation explained in § 2. The precepts of § 2, *d*, *e*, should be observed in computing the corrections. The tabulated values of the functions are generally given to four significant figures; but, in the tables of tangents and secants, they are sometimes given to a less number of figures (to avoid errors in interpolation), and are sometimes omitted altogether. In these cases the functions can be best found by finding their logarithms by the table of *Logarithms of Circular Functions* (see § 7), and then the numbers corresponding by the table of *Logarithms*.

b. To find any function of an angle *greater than* 90°, we must subtract from the given angle the greatest multiple of 90° which it contains; if an *even* multiple has been subtracted, we look out the required function of the remainder; if an *odd* multiple, the complementary function; and we then fix the *sign* of the function by considering the *quadrant* in which the given angle lies. For a *negative* angle, we find the required function of the corresponding positive angle, and then fix its sign by considering the quadrant of the angle.

c. Examples of the use of these tables: —

$\sin 77° 37' = 0.9767$, $\tan 53° 04' = 1.330$, $\sec 68° 45' = 2.759$,
$\cos 16° 19' = 0.9597$, $\operatorname{ctn} 3° 18' = 17.4$, $\csc 55° 13' = 1.217$;

$\sin 257° 37' = -0.9767$, $\tan 93° 18' = -17.4$, $\sec 325° 13' = 1.217$,
$\cos 163° 41' = -0.9597$, $\operatorname{ctn} 323° 04' = -1.330$, $\csc 158° 45' = 2.759$;

$\sin(-257° 37') = 0.9767$, $\tan(-93° 18') = 17.4$, $\sec(-325° 13') = 1.217$,
$\cos(-163° 41') = -0.9597$, $\operatorname{ctn}(-323° 04') = 1.330$, $\csc(-158° 45') = -2.759$;

$\sin^{-1} 0.2000 = 11° 32'$ or $168° 28'$ or $371° 32'$, etc.,
$\cos^{-1}(-0.3542) = 110° 45'$ or $249° 15'$ or $830° 45'$, etc.,
$\tan^{-1}(-4.570) = 102° 21'$ or $282° 21'$ or $-77° 39'$, etc.,
$\operatorname{ctn}^{-1} 0.3163 = 72° 27'$ or $252° 27'$ or $-107° 33'$, etc.,
$\sec^{-1} 5.000 = 78° 28'$ or $-78° 28'$ or $\pm 281° 32'$, etc.,
$\csc^{-1}(-3.529) = -16° 28'$ or $196° 28'$ or $-163° 32'$, etc.

§ 7. LOGARITHMS OF CIRCULAR FUNCTIONS.

To find the logarithm of any circular function of a given angle.

a. If the angle is *less than* 6°, the part of the table which occupies the upper half of p. 10 may be used. (See also *g*.) The *left-hand division* of this part of the table gives the values of a logarithm S (the characteristic and the first two figures of the mantissa being printed at the head of the column), with the angular limits between which each value may be used. Thus, for all positive angles less than 1° 51′.479, S = 6.4637; for all angles between 1° 51′.479 and 2° 49′.567, S = 6.4636; etc. The *next following*

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division gives, in like manner, the values of a logarithm T. We must find *the logarithm of the angle*, reduced to minutes and decimals of a minute, and must then apply the formulas: —

$$\log \sin \phi = \log (\phi \text{ in minutes}) + S - 10,$$
$$\log \tan \phi = \log (\phi \text{ in minutes}) + T - 10.$$

The two right-hand divisions of this part of the table give the values of the log sec, with the angular limits for each value. The logarithms of the cosine, cotangent, and cosecant are the *arithmetical complements* (— 10) of the logarithms of the secant, tangent, and sine, respectively. Example: —

l sin 3° 15′.23 = 8.7541, l tan 3° 15′.23 = 8.7548, l sec 3° 15′.23 = 0.0007,
l csc 3° 15′.23 = 1.2459, l ctn 3° 15′.23 = 1.2452, l cos 3° 15′.23 = 9.9993;

the negative characteristics being here, as in the following examples, made positive by the addition of 10.

b. If the angle is *acute and greater than* 84°, we must take its *complement*, and then seek the *function complementary to that required*, for the angle thus obtained, by the method just expounded. Example: —

l sin 86° 44′.77 = 9.9993, l tan 86° 44′.77 = 1.2452, l sec 86° 44′.77 = 1.2459,
l csc 86° 44′.77 = 0.0007, l ctn 86° 44′.77 = 8.7548, l cos 86° 44′.77 = 8.7541.

c. If the angle is contained *between* 6° *and* 84°, we use the main part of the table, occupying the lower half of p. 10 and pp. 11–15. The angle is tabulated at intervals of 10′, from 6° to 45° in the left-hand column of the table, and from 45° to 84° in the right-hand column. The names of the functions are to be taken from the *tops* of the columns, when the angle is on the *left*; and from the *bottoms* of the columns, when the angle is on the *right*. The angles on the right and left of any line and the names at the top and bottom of any column have the same relation to each other as in the tables of *Natural Values* (§ 6). The true characteristic in the first, third, and sixth columns is —1, but is printed 9. The six columns are arranged in pairs. The two functions in each pair of columns are *reciprocal* to each other; and the logarithms are therefore *complementary*, and their differences are equal in value, with opposite signs. Down the middle of each double column are printed, in small type, the *rates of difference* of the logarithms in that double column. Each value of this rate may be used in interpolation, instead of Δu, *through half the interval before and after the line on which it stands*, as stated in § 2, *c*. Thus, in finding the logarithms of the circular functions of any angle between 25° 25′ and 25° 35′ we work from the values corresponding to 25° 30′, the *nearest* tabulated angle; and compute the corrections by taking proportional parts of 26, 33, and 6, for the three pairs of functions. In applying the corrections, we must carefully observe, for each function, *whether the function is increasing or decreasing*.

For example, let the logarithms of the circular functions of 25° 27′.4 = 25° 30′ — 02′.6 be required. We find

l sin 25° 30′ = 9.6340	l tan 25° 30′ = 9.6785	l sec 25° 30′ = 0.0445
0.26 × 26 = 7	0.26 × 33 = 9	0.26 × 6 = 2
l sin 25° 27′.4 = $\overline{9.6333}$,	l tan 25° 27′.4 = $\overline{9.6776}$,	l sec 25° 27′.4 = $\overline{0.0443}$,

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l csc 25° 30′ = 0.3660	l ctn 25° 30′ = 0.3215	l cos 25° 30′ = 9.9555
7	9	2
l csc 25° 27′.4 = 0.3667,	l ctn 25° 27′.4 = 0.3224,	l cos 25° 27′.4 = 9.9557.

In like manner, we have

l sin 74° 46′ = 9.9845,	l tan 74° 46′ = 0.5649,	l sec 74° 46′ = 0.5804,
l csc 74° 46′ = 0.0155,	l ctn 74° 46′ = 9.4351,	l cos 74° 46′ = 9.4196.

d. If the angle is *greater than* 90°, or *negative*, we must use the method explained in § 6, *b*, for the tables of *Natural Values* of the circular functions. When the natural value of a circular function is negative, this should be indicated by writing the letter *n* after its logarithm. Examples:

l sin 105° 14′ = 9.9845, l tan 105° 14′ = 0.5649 *n*,
l sec 105° 14′ = 0.5804 *n*,

l csc 164° 46′ = 0.5804, l ctn 164° 46′ = 0.5649 *n*,
l cos 164° 46′ = 9.9845 *n*,

l sin (—74° 46′) = 9.9845 *n*, l tan (—105° 14′) = 0.5649,
l cos (—394° 46′) = 9.4196.

Given the logarithm of any circular function, to find the value of the corresponding angle.

e. If the given logarithm lies *without the limits of the main part of the table*, the upper part of p. 10 may be used. If the given logarithm is a *log sin less than* 9.0192, or a *log tan less than* 9.0216, subtract from it the proper value of S or T (or add the arithmetical complement), and the remainder is the log of the required angle in minutes. The limiting values of the log sin and log tan for each value of S and T are given in the table. If the given log is a *log csc greater than* 0.9808, or a *log ctn greater than* 0.9784, its arithmetical complement will be a log sin less than 9.0192, or a log tan less than 9.0216. If the given log is a *log sec less than* 0.0024, the limits between which the required angle lies are given by the table; the angle may have any value between these limits, and is not therefore very closely determined. If the given log is a *log cos greater than* 9.9976, its arithmetical complement is a log sec less than 0.0024.

If the given log is a *log sin, log tan, or log sec greater than* 9.9976, 0.9784, *or* 0.9808 (respectively), or a *log csc, log ctn, or log cos less than* 0.0024, 9.0216, *or* 9.0192 (respectively), we must change the name of the function to the *complementary name* (sin to cos, etc.), *find the corresponding angle* as above, and take the *complement* of the angle thus found. Examples:—

(log sin)$^{-1}$ 8.9542 = 5° 09′.8, (log ctn)$^{-1}$ 2.0531 = 0° 30′.42,
(log ctn)$^{-1}$ 9.0024 = 84° 15′.5, (log sin)$^{-1}$ 9.9983 = 84° 56′ ± 4½′.

f. If the given logarithm is contained *within the limits of the main part of the table*, the required angle is found by ordinary interpolation; and we may use the printed rate of difference as the value of Δu, working in each case from the *nearest* tabulated value. The angle should be found to the *nearest minute*, or, when the difference exceeds 100, to the nearest *tenth* of a

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minute. But in the right-hand pair of columns, the last figure of the angle thus found will generally be uncertain. Examples: —

Let it be required to find $(\log \sec)^{-1}\, 0.0643$; *i. e.* the angle of which the log sec is 0.0643. The nearest tabulated log sec is 0.0647. We have, then,

$$(\log \sec)^{-1}\, 0.0647 = 30^\circ\, 30',\ u_2 - u = 4,\ \Delta u = 7,\ \frac{4}{7} = 0.6,$$

$$\therefore\ (\log \sec)^{-1}\, 0.0643 = 30^\circ\, 30' - 06' = 30^\circ\, 24'.$$

In like manner, let the student find

$(\log \sin)^{-1}\, 9.5663 = 21^\circ\, 37'$, $(\log \operatorname{ctn})^{-1}\, 0.0496 = 41^\circ\, 44'$,
$(\log \cos)^{-1}\, 9.9188 = 33^\circ\, 58'$ or $57'$, $(\log \sec)^{-1}\, 0.2272 = 53^\circ\, 39'$,
$(\log \tan)^{-1}\, 0.7507 = 79^\circ\, 56'$, $(\log \csc)^{-1}\, 0.1433 = 45^\circ\, 59'$.

The angle may also be found by the next following table.

g. Pp. 8 and 9 may also be used for angles less than 6° or greater than 84°.

E.g. $1 \sin 4^\circ\, 03'.4 = 8.8497$, $1 \tan 4^\circ\, 03'.4 = 8.8508$, $1 \sec 4^\circ\, 03'.4 = 0.0011$,
$1 \csc 4^\circ\, 03'.4 = 1.1503$, $1 \operatorname{ctn} 4^\circ\, 03'.4 = 1.1492$, $1 \cos 4^\circ\, 03'.4 = 9.9989$.

§ 8. INVERSE CIRCULAR FUNCTIONS.

a. The table having this heading (pp. 16–18) is a table for finding the angle which corresponds to the given logarithm of a circular function. The logarithm (increased by 10) is the *argument* of the table, and is to be regarded as given to *four places* of decimals. It is tabulated at intervals of 0.0100 from 9.0000 to 0.0000 through the first page of the table, then at intervals of 0.0010, and in the last two divisions at intervals of 0.0001. The characteristic of the argument is printed at the head of the column. The figures supposed to follow the printed figures in the values of the argument are *zeros*. Thus, the first value is 9.0000, the next 9.0100, etc. The angle is given, for convenience of interpolation, in degrees and decimals of a degree. When found, it is easily reduced to degrees and minutes, if that is necessary, and should, in general, be taken *only to the nearest minute*. The angle under the heading $\sin^{-1} u$ is that angle of which the corresponding value of the argument, $\log u$, is the log sin; etc. In interpolating in this table, we may use the *printed rate of difference* instead of Δu, working from the *nearest* tabulated value of the argument, and carefully observing whether the tabulated angle ought to be *increased* or *diminished*. *When the printed rate of difference is omitted*, this is because the interval is too great to admit of accurate interpolation. In this case, we must resort to those later divisions of the table in which the argument is tabulated at smaller intervals. *When the last figure of the tabulated angle is printed in small type*, this shows that that figure is uncertain, if the logarithm is given to only four places; that is, that there is a possible variation, on each side of the tabulated angle, as great as *half a unit* in the place of the figure so printed. For example, if $\log u = 9.9000$, we find the last figures of $\sin^{-1} u$ and $\cos^{-1} u$ to be printed in small type. Now, seven-place tables show that $(\log \sin)^{-1}\, 9.8999500 = 52°.581$, while $(\log \sin)^{-1}\, 9.9000500 = 52°.600$. But 9.9000 may represent any logarithm between these; and hence the corresponding angle, in this case, admits a like variation, while $\cos^{-1} u$ may have any value between 37°.419 and 37°.400.

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Neither of these difficulties presents itself in finding an angle from its log tan or log ctn. If $\log u = 9.9000$, $\tan^{-1} u$ can only vary from 38°.458 to 38°.464.

The angle found by interpolation should be carried out *only to the nearest hundredth of a degree, in any case.* The last column of the table shows that the angle is not always determined even to the nearest *tenth.*

b. If the characteristic of the given logarithm is 0, we must take its *arithmetical complement,* which will be the logarithm of the *reciprocal* function of the same angle. The angle can then be found by the table.

c. If the given logarithm is less than 9.0000, or greater than 0.0000, the tables in the upper part of p. 10 may be used, as explained in § 7, *e*; or pp. 8, 9.

d. Let us find by this table the angles sought above, in § 7, *f.* We have, in the case of the first example,

$$(\log \sec)^{-1}\, 0.0643 = (\log \cos)^{-1}\, 9.9357.$$

Then the table gives

$$(\log \cos)^{-1}\, 9.9360 = 30°.35, \qquad \Delta u = 23,$$
$$0.3 \times 0.23 = .07$$
$$\therefore\ (\log \cos)^{-1}\, 9.9357 = 30°.42 = 30°\ 25'.$$

In fact, the limits of the angle are 30° 24′.2 and 30° 25′.6, the mean value being 30° 24′.9. In this case, the present table gives a better value than the other; but both values are admissible.

In like manner, we have

$$(\log \sin)^{-1}\, 9.5663 = 21°.81 - 20 = 21°.61 = 21°\ 37',$$
$$(\log \cos)^{-1}\, 9.9188 = 33°.92 + .04 = 33°.96 = 33°\ 58',$$
$$(\log \tan)^{-1}\, 0.7507 = 79°.92 + .02 = 79°.94 = 79°\ 56',$$
$$(\log \operatorname{ctn})^{-1}\, 0.0496 = 41°.71 + .03 = 41°.74 = 41°\ 44',$$
$$(\log \sec)^{-1}\, 0.2272 = 53°.64 + .02 = 53°.66 = 53°\ 40',$$
$$(\log \csc)^{-1}\, 0.1433 = 46°.01 - .04 = 45°.97 = 45°\ 58'.$$

§ 9. HYPERBOLIC FUNCTIONS.

a. The **hyperbolic functions** are certain functions which bear relations to the equilateral hyperbola similar to those borne by the circular functions to the circle; and they may often be usefully employed both in computation and in analysis. They are named the hyperbolic sine, cosine, tangent, cotangent, secant, and cosecant; and are variously denoted by different writers. They are here represented by the symbols: Sh, Ch, Th, Cth, Sch, Csch. They may be defined by the following formulas, in which

$$\mathcal{G} = \text{the exponential base}$$
$$= 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots.$$
$$= 2.7182818285 \dots .:-$$

$$\operatorname{Sh} x = \tfrac{1}{2}(\mathcal{G}^{x} - \mathcal{G}^{-x}), \qquad \operatorname{Ch} x = \tfrac{1}{2}(\mathcal{G}^{x} + \mathcal{G}^{-x}), \qquad \operatorname{Th} x = \frac{\operatorname{Sh} x}{\operatorname{Ch} x},$$

$$\operatorname{Cth} x = \frac{1}{\operatorname{Th} x}, \qquad \operatorname{Sch} x = \frac{1}{\operatorname{Ch} x}, \qquad \operatorname{Csch} x = \frac{1}{\operatorname{Sh} x}.$$

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They bear to the circular functions the relations expressed by the following formulas, in which $i = \sqrt{-1}$: —

$$\text{Sh } x = \frac{\sin xi}{i}, \qquad \sin x = \frac{\text{Sh } xi}{i},$$
$$\text{Ch } x = \cos xi, \qquad \cos x = \text{Ch } xi,$$
$$\text{Th } x = \frac{\tan xi}{i}, \qquad \tan x = \frac{\text{Th } xi}{i},$$
$$\text{Cth } x = i \operatorname{ctn} xi, \qquad \operatorname{ctn} x = i \text{ Cth } xi,$$
$$\text{Sch } x = \sec xi, \qquad \sec x = \text{Sch } xi,$$
$$\text{Csch } x = i \operatorname{csc} xi, \qquad \operatorname{csc} x = i \text{ Csch } xi.$$

Again, if ϕ is so taken that

$$x = \text{nat log tan } (45^\circ + \tfrac{1}{2}\phi),$$

$$\text{Sh } x = \tan \phi, \qquad \text{Ch } x = \sec \phi, \qquad \text{Csch } x = \operatorname{ctn} \phi,$$
$$\text{Th } x = \sin \phi, \qquad \text{Sch } x = \cos \phi, \qquad \text{Cth } x = \operatorname{csc} \phi.$$

The value of ϕ determined by this formula has been called by some writers the **Gudermannian** of x, and denoted by the symbol: gd x.

b. From $x = 0.00$ to $x = 1.00$, the function tabulated is gd x in degrees, at intervals of 0.01 in the value of x. The hyperbolic functions of x are then readily found, by the aid of the formulas last given, from the tables of circular functions. Beginning with $x = 1.00$, log Sh x, log Ch x, and log Th x are tabulated, at intervals of 0.01 in the value of x, up to $x = 3.00$, the characteristic of each logarithm being placed at the head of its column; then at intervals of 0.1 up to $x = 6.0$; and lastly at intervals of 1 up to $x = 10.0$. The printed differences are to be used, as in other tables, each through *half the interval* before and after the line on which it stands.

c. If $x > 10$, log Th $x = 0.0000$, while log Sh x and log Ch x may be found by the formula and table given at the lower right-hand corner of p. 21. The quantity μ is the *modulus* of the denary system of logarithms; that is, it is the denary logarithm of the exponential base. The values of $n\mu$ being given for all integral values of n from 1 to 10, any product $x\mu$ is readily found, by adding together the products of μ by the successive figures of x. Only four decimal-figures should be retained in the result.

d. The functions log Cth x, log Sch x, and log Csch x are the *arithmetical complements* of log Th x, log Ch x, and log Sh x, respectively.

e. The table may be used both directly and inversely. Examples: —

log Sh 0.5378 = log tan 29°.427 = 9.7513,
log Ch 1.5280 = 0.3825,
log Cth 1.8240 = 0.0226,
log Sh 12.5913 = log Ch 12.5913 = 5.1673,
$(\log \text{Ch})^{-1}$ 1.6000 = 4.377,
$(\log \text{Sh})^{-1}$ 5.0000 = 12.206,
$(\log \text{Th})^{-1}$ 9.9012 = 1.089,
$(\log \text{Sch})^{-1}$ 9.5873 = 1.604,
$(\log \text{Csch})^{-1}$ 0.3924 = gd^{-1} $(\log \tan)^{-1}$ 9.6076 .
= gd^{-1} 22°.06 = 0.395.

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§ 10. NATURAL LOGARITHMS.

a. The **natural** system of logarithms is that which is founded on the **exponential base** (see § 9). This number is defined as *the limiting value* to which the expression

$$(1+\epsilon)^{\frac{1}{\epsilon}} = \sqrt[\epsilon]{(1+\epsilon)}$$

approaches, as ϵ approaches 0. It is most frequently denoted by the letter e; but, as being one of the few peculiar constants of analysis, it is here represented by the symbol $\mathfrak{G}$, which may be read "**base.**"

The following formulas are proved in treatises on the Differential Calculus: —

$$\mathfrak{G} = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots,$$

$$\mathfrak{G}^x = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \frac{x^4}{1.2.3.4} + \dots,$$

$$\text{nat log } (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots;$$

the second formula being applicable to *all values* of x, but the last only when x is *numerically less than* 1. If x is *very small*, then *approximately*

$$\mathfrak{G}^x = 1 + x, \qquad \text{nat log } (1+x) = x, \qquad \text{nat log } (1-x) = -x.$$

We also have, in the natural system,

$$\log (a+h) = \log a + \log\left(1 + \frac{h}{a}\right) = \log a + \frac{h}{a} - \frac{h^2}{2a^2} + \frac{h^3}{3a^3} - \dots,$$

provided h is numerically less than a.

The *rate of difference* of $\mathfrak{G}^x$, for $\Delta x = 1$, is always $\mathfrak{G}^x$, and that of nat log x is $\frac{1}{x}$.

b. The **numerical value** of $\mathfrak{G}$ or of any power of $\mathfrak{G}$ can be computed, to *any assigned number of decimal-places*, by using a sufficient number of terms of the first two series given above. Thus, to find $\mathfrak{G}$ to four decimal-places, we proceed as follows, observing that, if any term be divided by its number in the series, the next following term is obtained: —

1)	1.00000
2)	1.00000
3)	0.50000
4)	0.16667
5)	0.04167
6)	0.00833
7)	0.00139
8)	0.00020
	0.00002
$\mathfrak{G}$	= 2.7183 .....

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c. The **modulus** of any system of logarithms is the logarithm of 6 in that system. If m is the modulus of a system of which a is the base, then

$$a^m = 6, \qquad 6^{m^{-1}} = a.$$

The modulus of the *natural system itself* is 1. The values of the modulus of the *denary system* and of the reciprocal of that modulus are

$$\mu = \text{den log } 6 = 0.4342944819\ldots,$$
$$\mu^{-1} = \text{nat log } 10 = 2.3025850930\ldots.$$

By the rule for converting logarithms from one system to another, the logarithm of a number in any system may be found by *multiplying the modulus of that system into the natural logarithm of the same number.* Thus,

$$\text{den log } x = \mu \text{ nat log } x,$$
$$\text{nat log } x = \mu^{-1} \text{ den log } x.$$

By the aid of these formulas, the table at the bottom of p. 21 may be used to find the natural logarithm of any number, or the denary logarithm of any power of the exponential base, or to find a number from its natural logarithm. For example:—

$$\text{nat log } 72.5 = 1.8603 \times \mu^{-1} = 4.2835,$$
$$\text{nat log } 1.0074 = 0.0032 \times \mu^{-1} = 0.0074,$$
$$\text{den log } 6^{\frac{1}{7}} = \tfrac{1}{7}\mu = 0.0620,$$
$$(\text{nat log})^{-1}\, 10.2108 = (\text{den log})^{-1}\, (10.2108 \times \mu)$$
$$= (\text{den log})^{-1}\, 4.4345 = 27194.$$

d. The natural system is so called, because, in the higher mathematics, it is convenient to regard all other systems as founded upon this. It is named by some writers **hyperbolic**, and by others **Neperian**. But in fact, it is not the system of Napier; nor has it any other relation to the hyperbola than that which belongs to logarithms in general.

e. We may make the following statement of the relation of logarithms and of the hyperbolic functions to the hyperbola, using the notation of Analytic Geometry:—

Let $xy = 1$ be the equation of an hyperbola referred to its asymptotes. It can be proved by the Integral Calculus that the *area*, contained between the curve and the axis of x, and between two ordinates of which one is drawn to the vertex of the curve, is measured by log x in the system of which the modulus is $\sin \omega$. Thus, *the logarithms belonging to any system may be represented by the areas of an appropriate hyperbola.* The natural system corresponds to the equilateral hyperbola, for which $\sin \omega = 1$.

Again, if u denotes twice the area of the *sector* of the hyperbola $x^2 - y^2 = 1$, contained between the axis of x and a radius vector from the centre, then

$$x = \text{Ch } u, \qquad y = \text{Sh } u;$$

just as, in the circle $x^2 + y^2 = 1$, with a similar meaning of u,

$$x = \cos u, \qquad y = \sin u.$$

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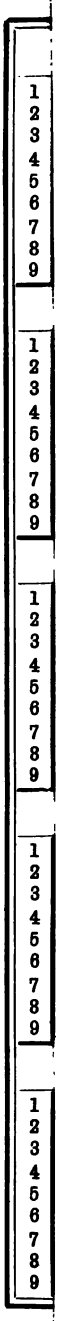

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